

Under the Robotic Knife: A Verifiable Controller for use of Multiple Robotic Arms in Surgery



Multi-Arm Surgical Robotics

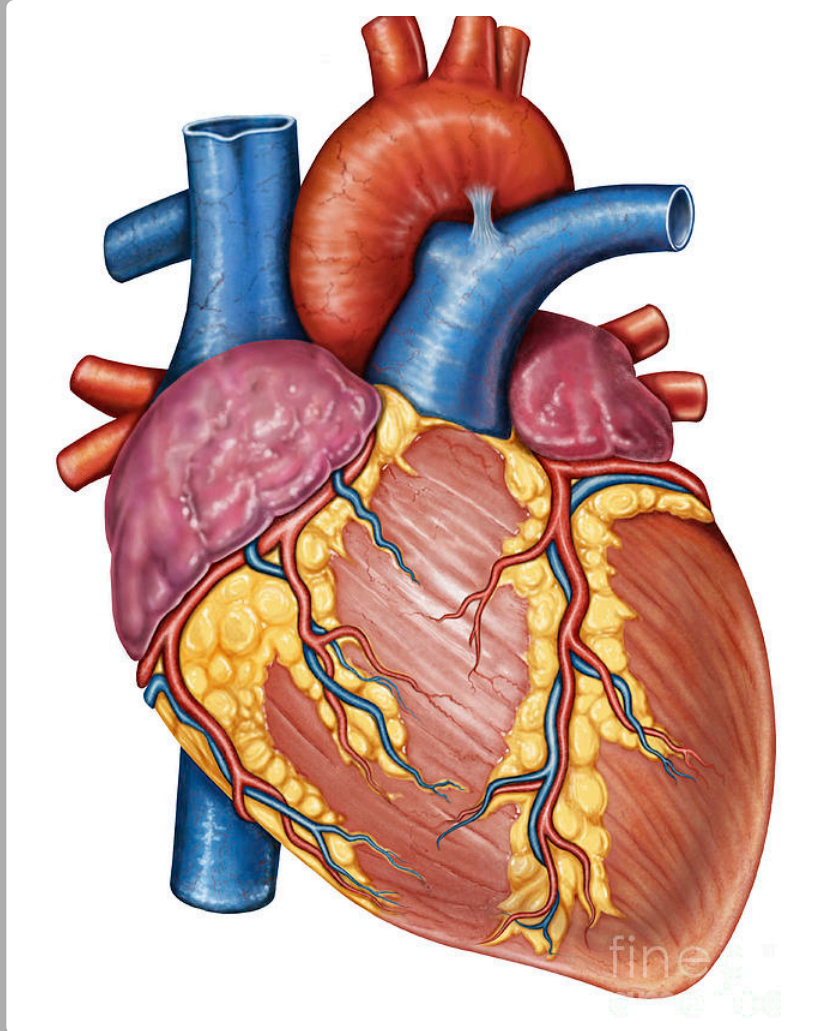
Problem

Models

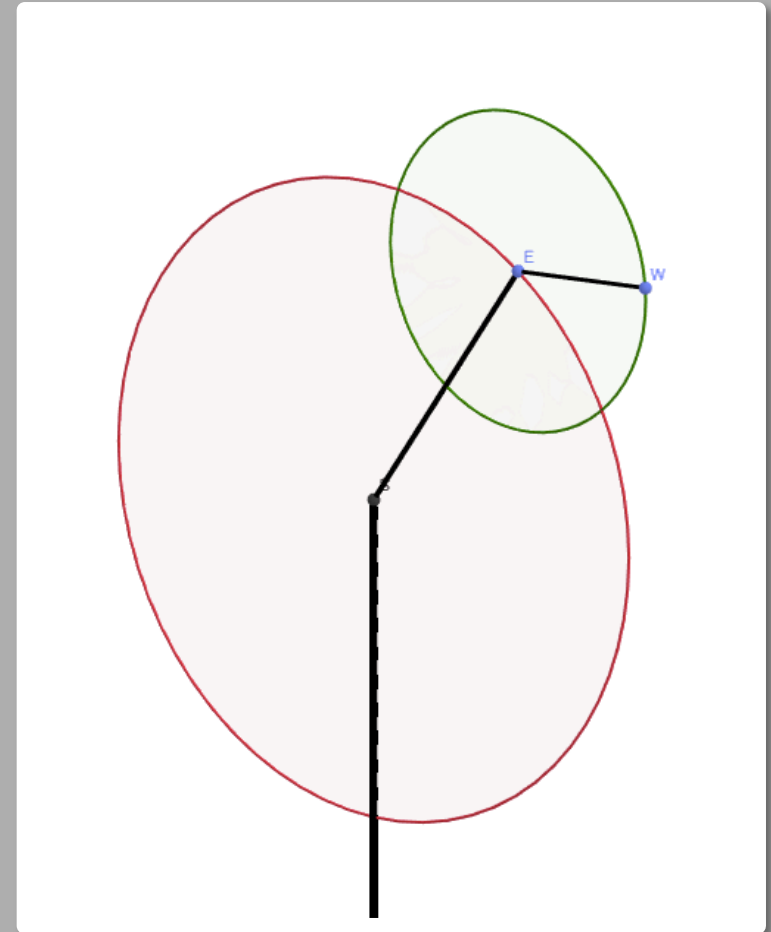
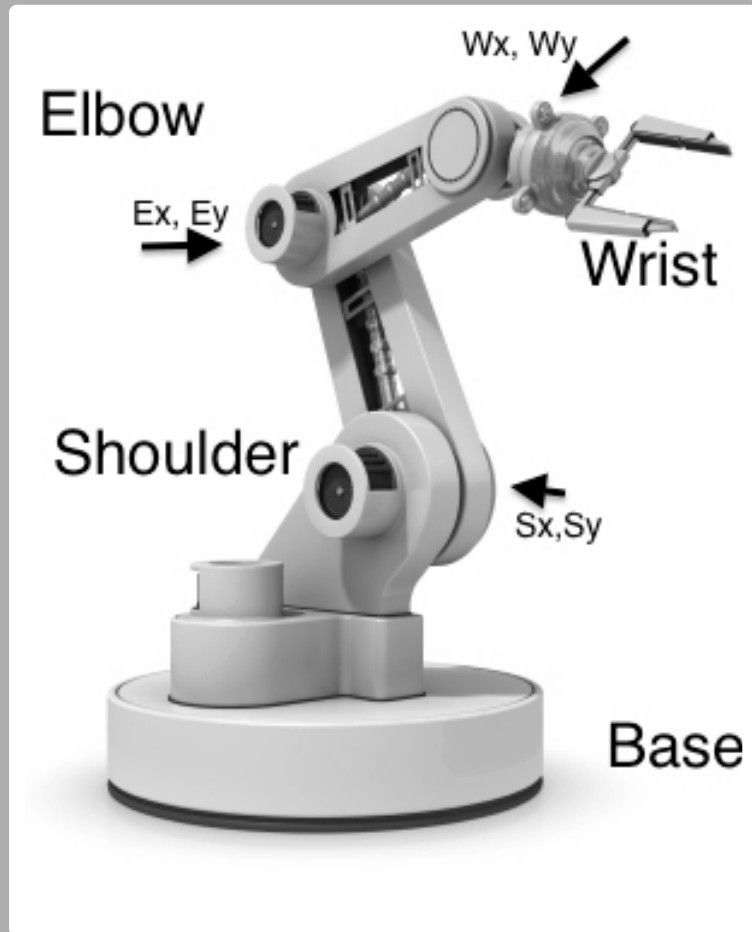
Conclusion



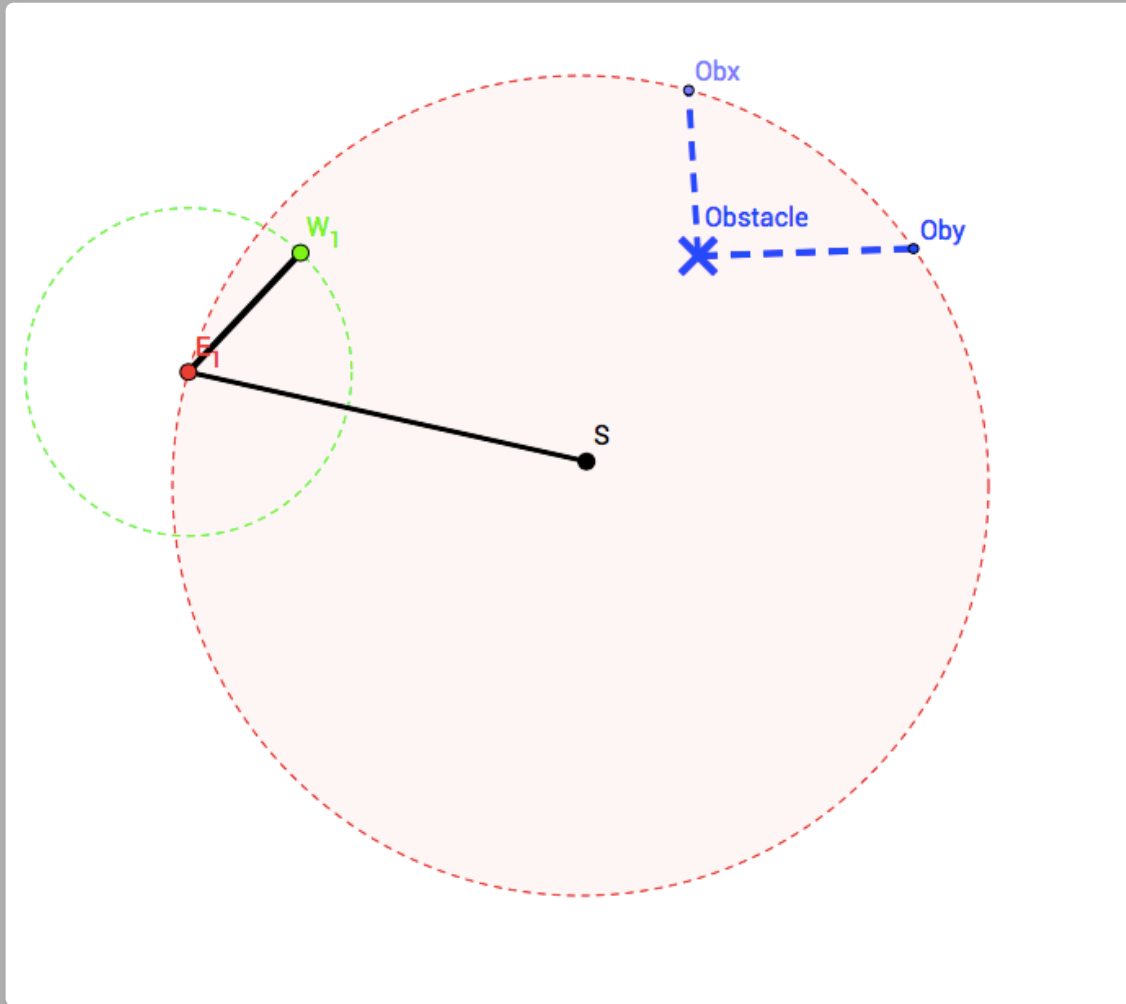
What can go wrong?



Representation:



Static Obstacle Avoidance:

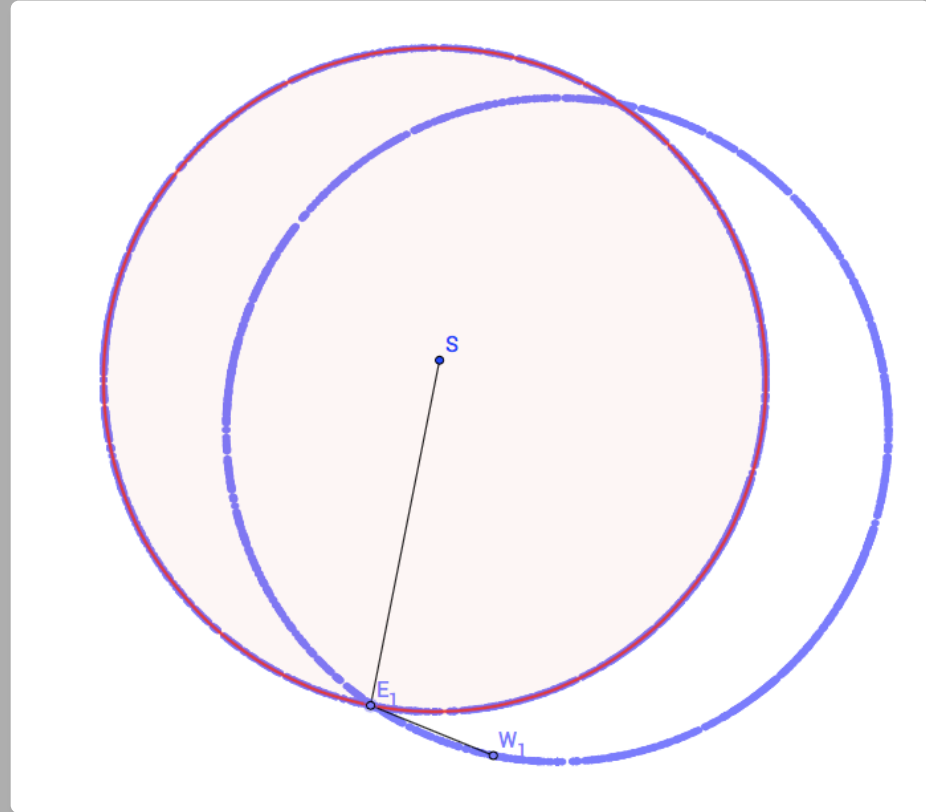


Dynamics, Safety, Control:

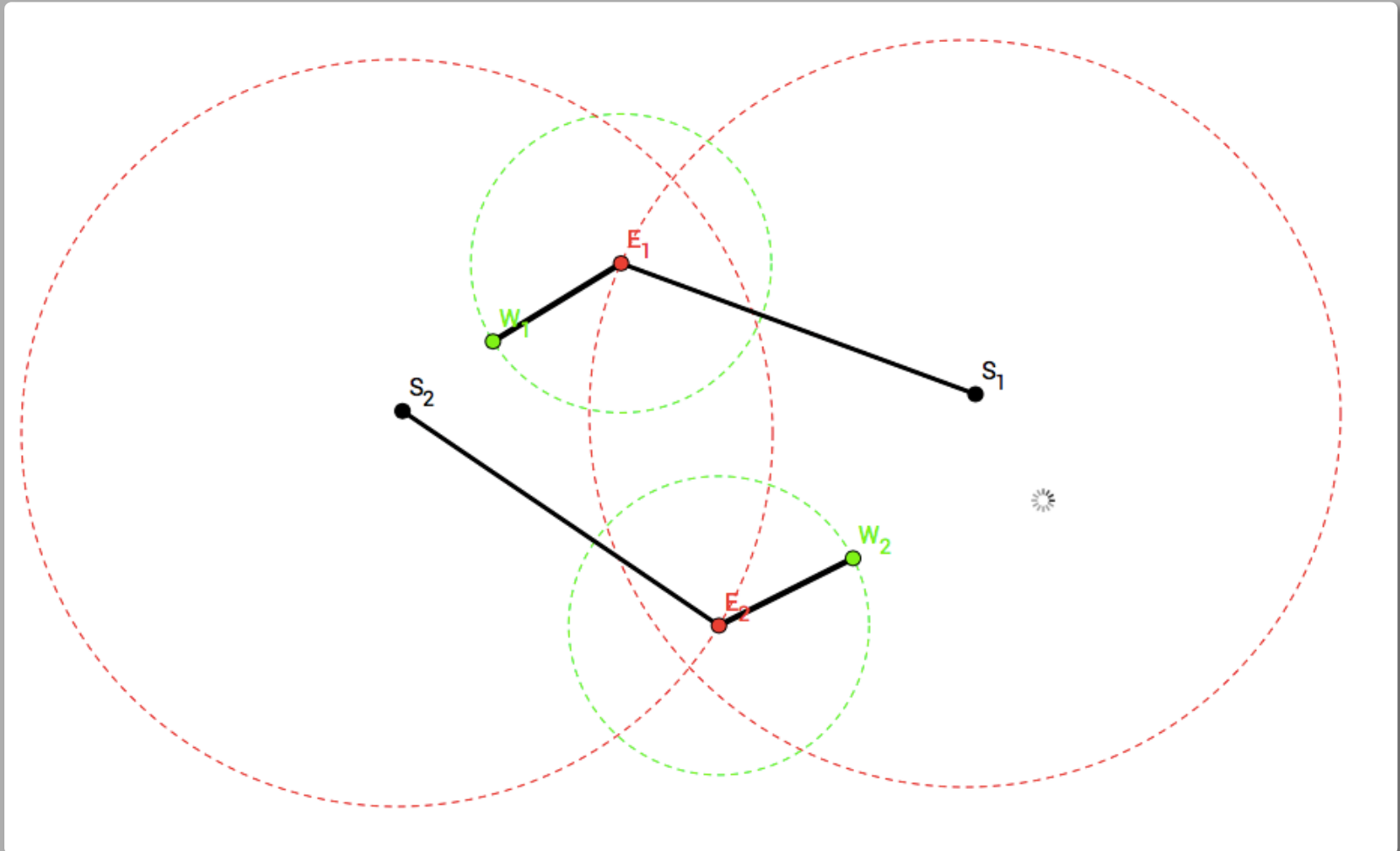
$$\begin{aligned} & \forall c. (c \geq 0 \wedge c \leq 1 \rightarrow \\ & (c * ex \neq obx \vee c * ey \neq oby)) \wedge \\ & \forall c. (c \geq 0 \wedge c \leq 1 \rightarrow \\ & (c * rwx + ex \neq obx \vee c * rwy + ey \neq oby)) \end{aligned}$$

$$wx := ex + rwx$$

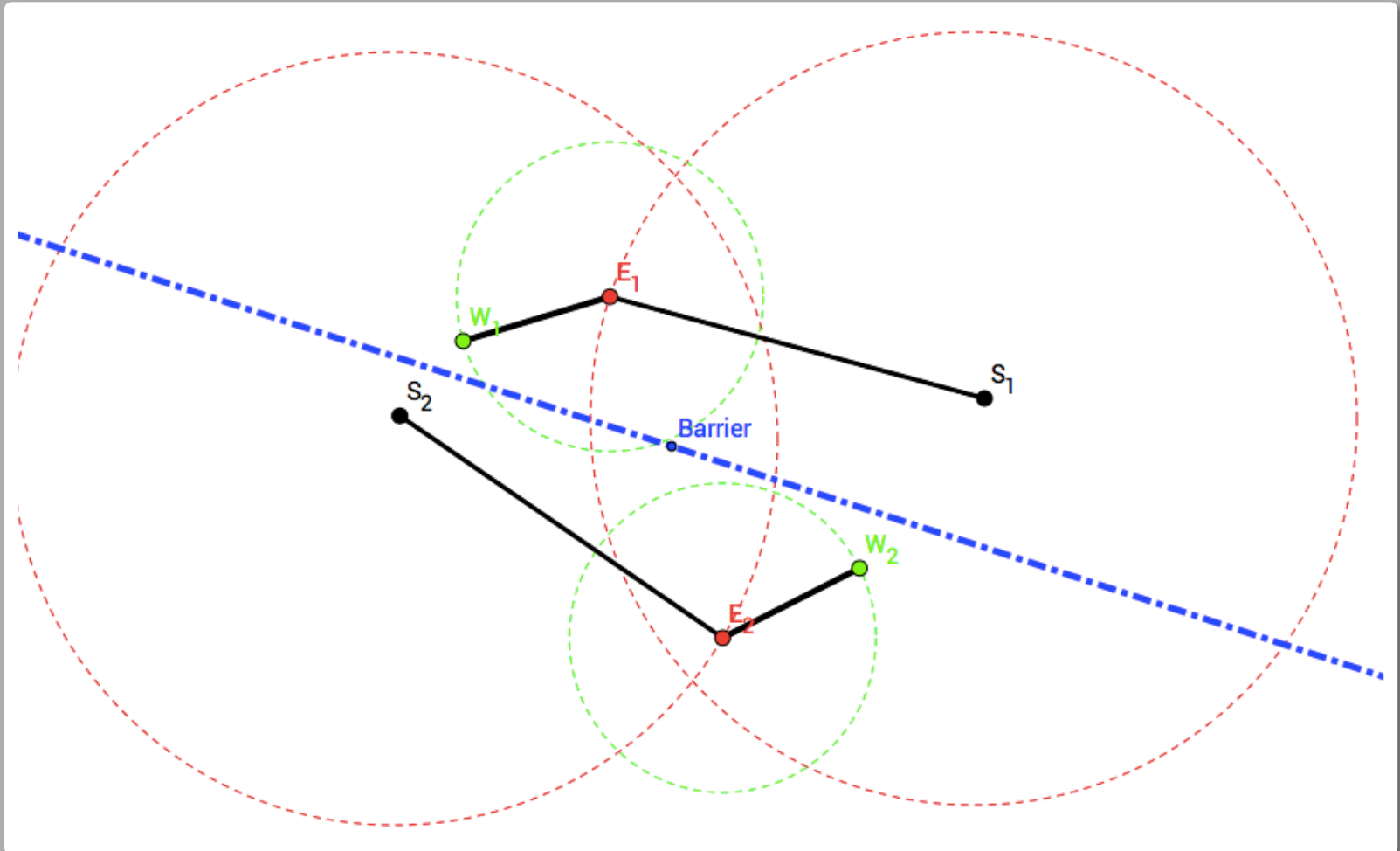
$$wy := ey + rwy$$



Robotic Arm Intersection:

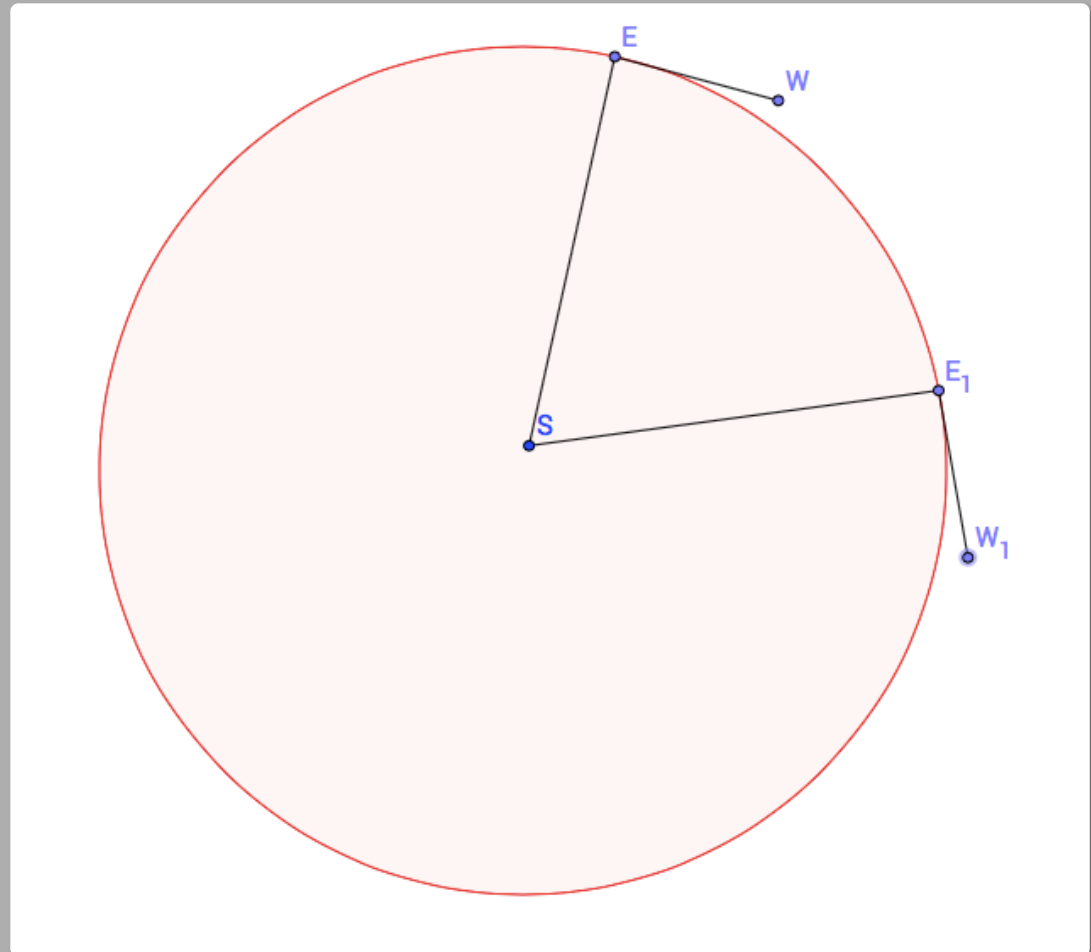


Robotic Arm Intersection:



Dynamics:

```
{ex' = ve*dex,  
ey' = ve*dey,  
rwx' = vw*dex,  
rwy' = vw*dwy,  
dex' = -ve*dey/re,  
dey' = ve*dex/re,  
dwx' = -vw*dwy/rw,  
dwy' = vw*dwx/rw,  
t' = 1  
& t <= T}
```

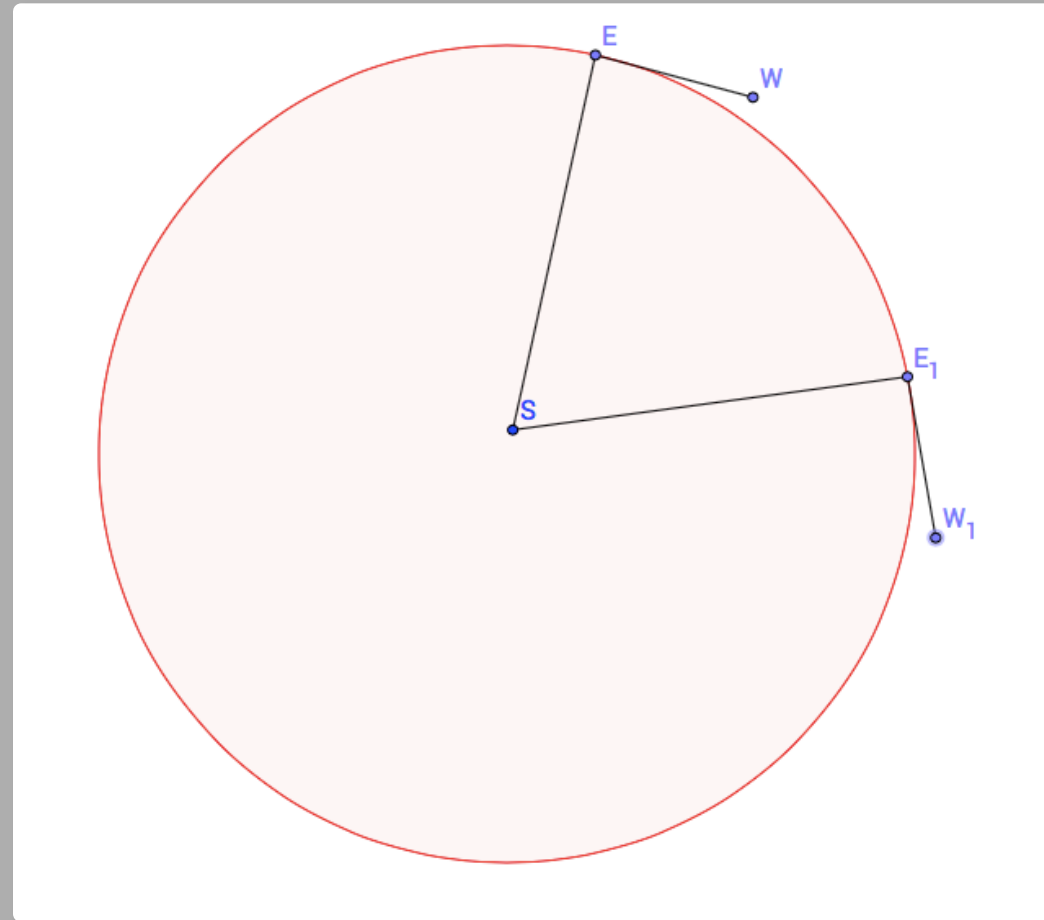


```
cos := dwx*dex+dwy*dey;  
sin := dwx*dey-dwy*dex;
```

```
wx:=ex+rwx+rw*(dex*cos+dey*sin);  
wy:=ey+rwy-rw*(-dex*sin+dey*cos)
```

Dynamics:

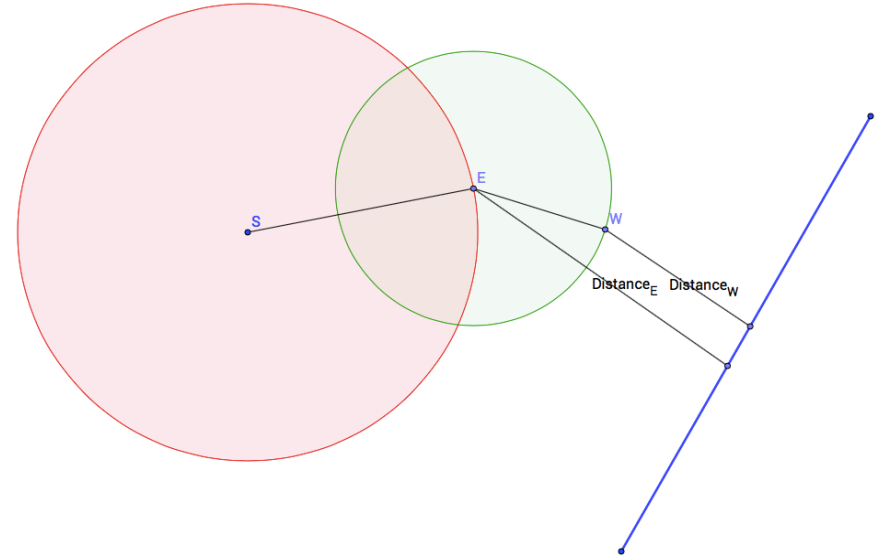
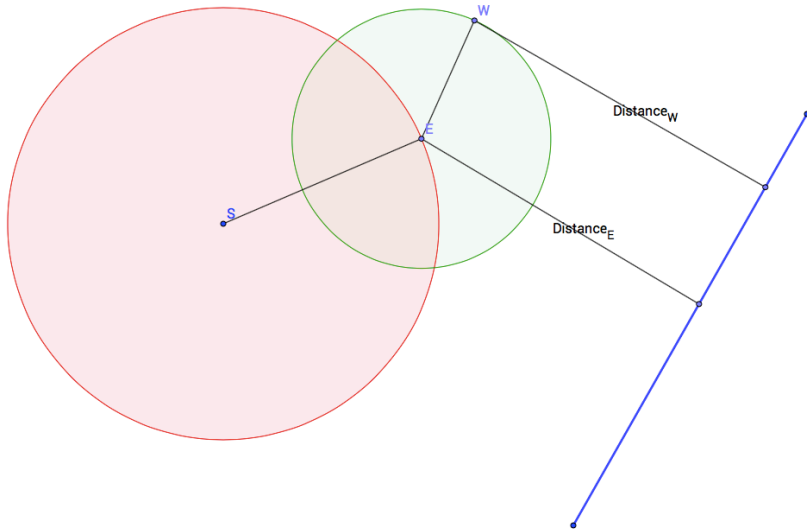
$$\begin{bmatrix} dwx \\ dwy \end{bmatrix} = \begin{bmatrix} \cos & \sin \\ -\sin & \cos \end{bmatrix} \begin{bmatrix} dex \\ dey \end{bmatrix}$$



```
cos := dwx*dex+dwy*dey;  
sin := dwx*dey-dwy*dex;
```

```
wx:=ex+rx+rw*(dex*cos+dey*sin);  
wy:=ey+ry-rw*(-dex*sin+dey*cos)
```

Safety and Control:



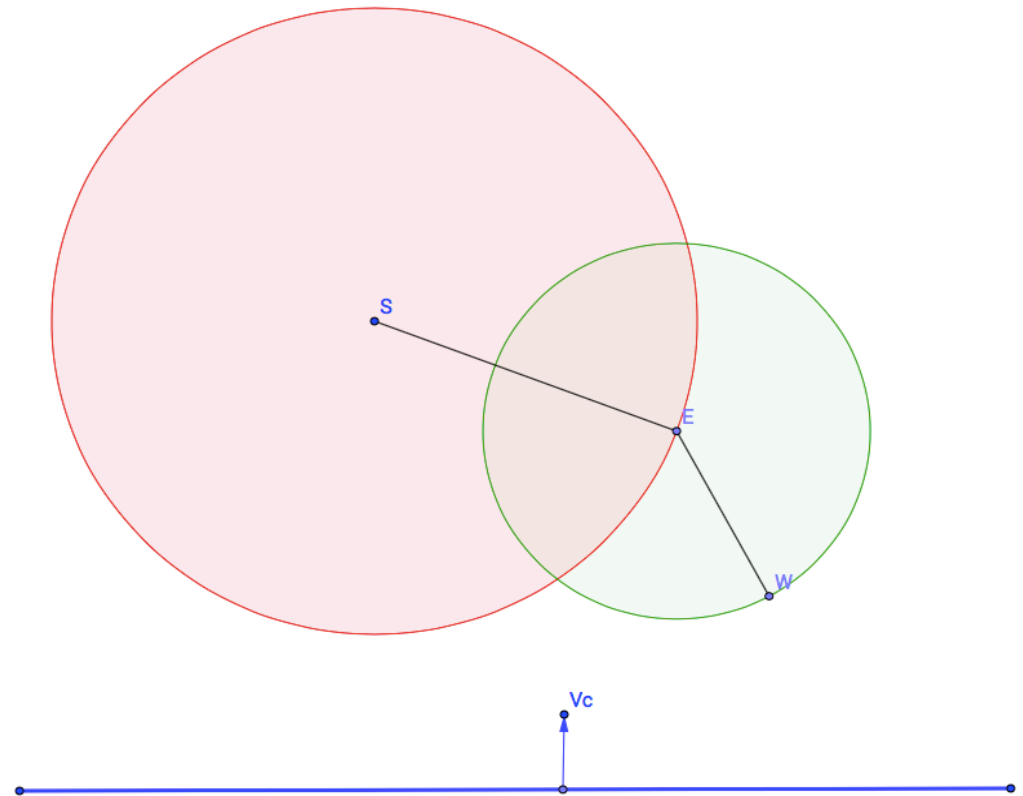
$$D(x_0, y_0) = \frac{a * x_0 + b * y_0 + c}{(a^2 + b^2)^{(1/2)}}$$

$$(aey + bex)^2 \leq (a^2 + b^2) * re^2$$

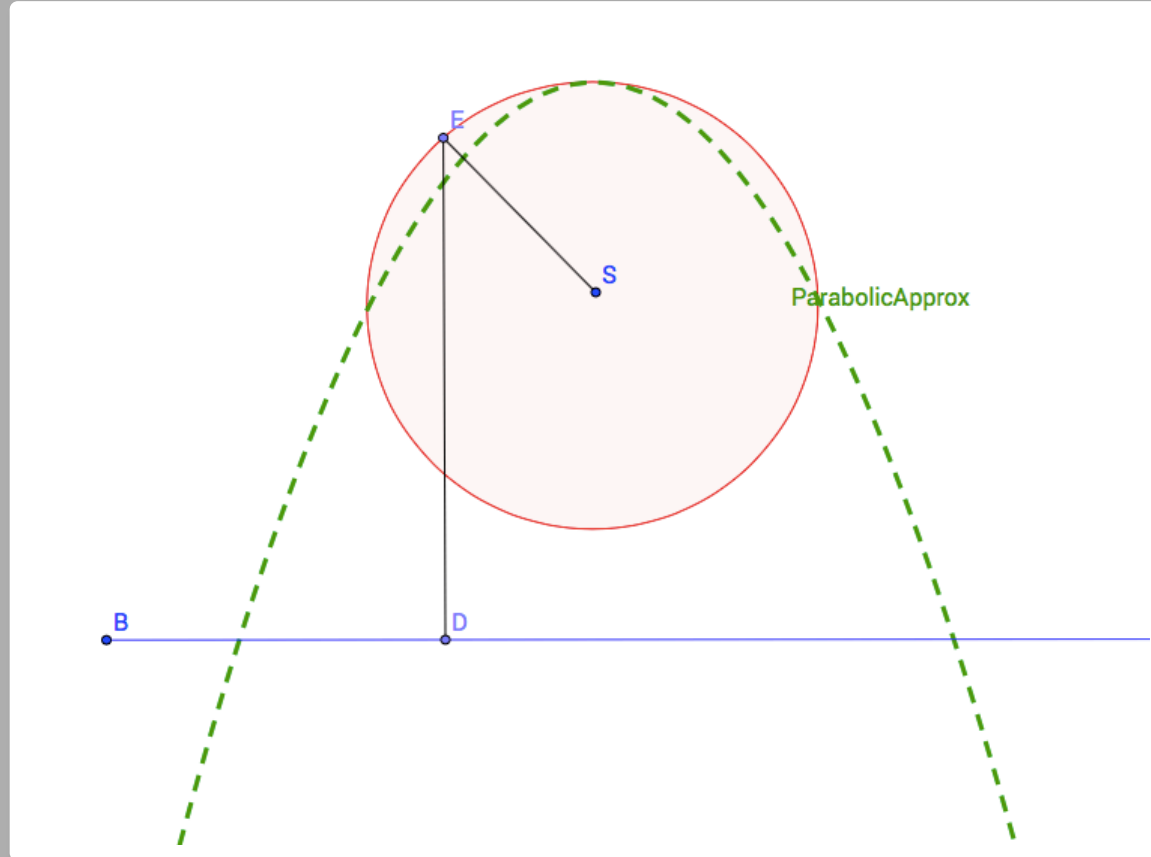
Dynamic Barrier:

```
{ex' = -ve*ey/re,  
 ey' = ve*ex/re,  
 rwx' = -vw*rwy/rw,  
 rwy' = vw*rwx/rw,  
 c' = -vc,  
 t' = 1  
 & t <= T  
 & (ey <= 0 |  
 wy <= 0) }
```

$$Vc = b * Ex * ve/re.$$

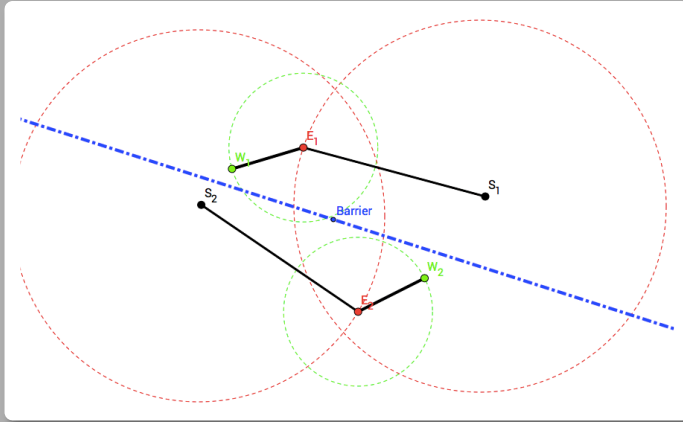


Parabolic Approximation:



$$Y_{approx} = (X_{approx})^2 / re + re$$

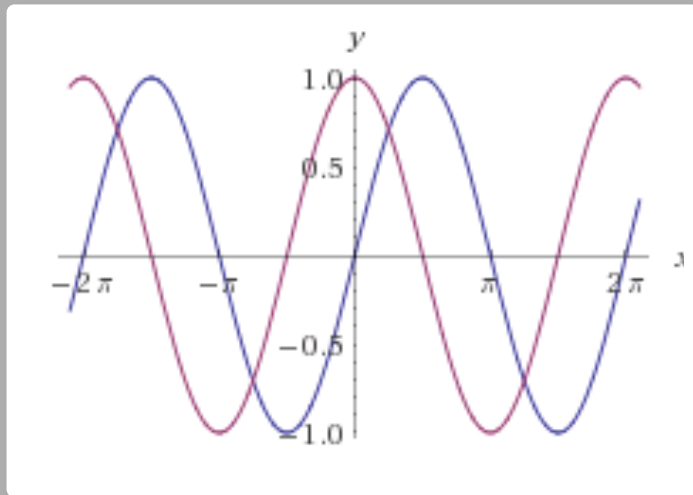
$$X_{approx} = X0 - Ve * t$$



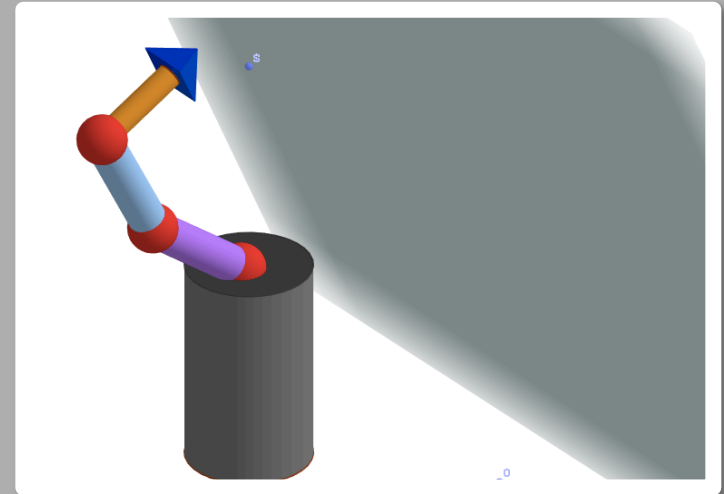
Accomplishments:



Applications:



Challenges:



Future Work:

Questions?

