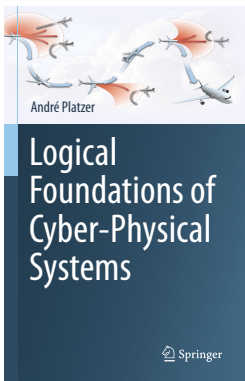
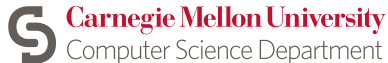


09: Reactions & Delays

Logical Foundations of Cyber-Physical Systems



André Platzer





- 1 Learning Objectives
- 2 Delays in Control
 - The Impact of Delays on Event Detection
 - Cartesian Demon
 - Model-Predictive Control Basics
 - Design-by-Invariant
 - Controlling the Control Points
 - Sequencing and Prioritizing Reactions
 - Time-Triggered Verification
- 3 Summary

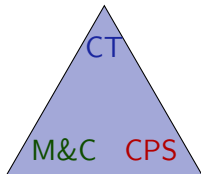
1 Learning Objectives

2 Delays in Control

- The Impact of Delays on Event Detection
- Cartesian Demon
- Model-Predictive Control Basics
- Design-by-Invariant
- Controlling the Control Points
- Sequencing and Prioritizing Reactions
- Time-Triggered Verification

3 Summary

using loop invariants
design time-triggered control
design-by-invariant



modeling CPS
designing controls
time-triggered control
reaction delays
discrete sensing

semantics of time-triggered control
operational effect
finding control constraints
model-predictive control



1 Learning Objectives

2 Delays in Control

- The Impact of Delays on Event Detection
- Cartesian Demon
- Model-Predictive Control Basics
- Design-by-Invariant
- Controlling the Control Points
- Sequencing and Prioritizing Reactions
- Time-Triggered Verification

3 Summary

Quantum's Ping-Pong Proof Invariants

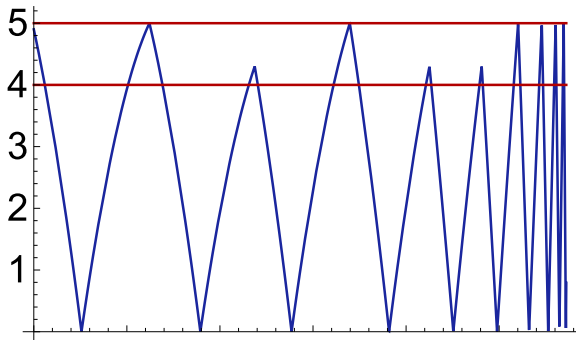
Proposition (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[\{x' = v, v' = -g \wedge x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \wedge x \geq 5\});$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv]^*(0 \leq x \leq 5)$$

Proof **@invariant**($0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$)



Quantum's Ping-Pong Proof Invariants

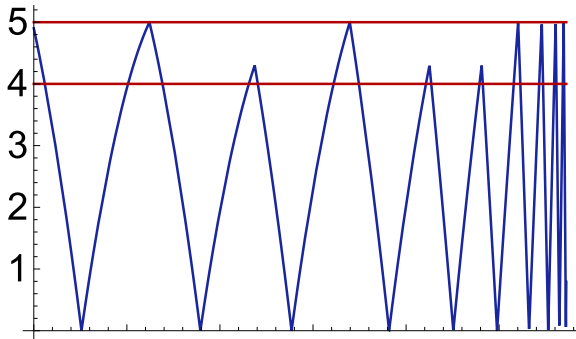
Proposition (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[\{x' = v, v' = -g \wedge x \geq 0 \wedge x \leq 5\} \cup \{x' = v, v' = -g \wedge x \geq 5\}];$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv)^*(0 \leq x \leq 5)$$

Proof **@invariant**($0 \leq x \leq 5 \wedge (x = 5 \rightarrow v \leq 0)$)



Just can't implement ...

Physical vs. Controller Events

- 1 Justifiable: Physical events (on ground $x = 0$)
- 2 Justifiable: Physical evolution domains (above ground $x \geq 0$)
- 3 Questionable: Controller evolution domain ($x \leq 5$)
- 4 Unlike physics, controllers won't run *all* the time. Just fairly often.
- 5 Controllers cannot sense and compute all the time.

If you expect the world to change for your controller's sake, you may be in for a surprise.

Conjecture (Quantum can play ping-pong safely)

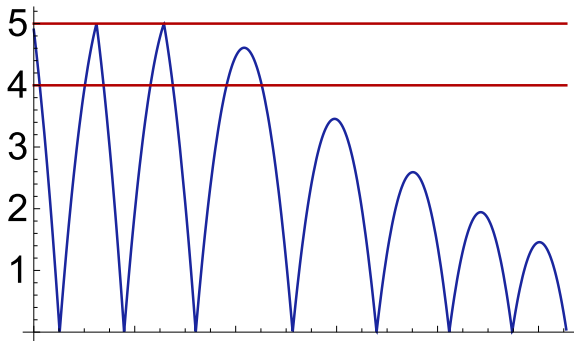
$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\{x' = v, v' = -g \ \& \ x \geq 0\};$$

$$\text{if}(x=0) \ v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv)^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes



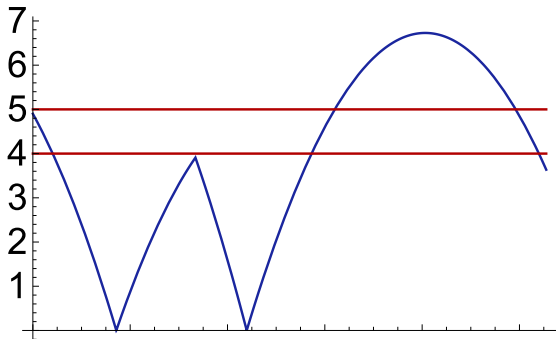
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\{x' = v, v' = -g \ \& \ x \geq 0\};$$

$$\text{if}(x=0) \ v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv)^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



Could miss if-then event

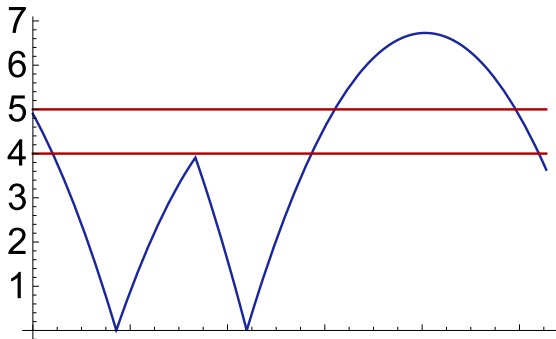
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\{x' = v, v' = -g \ \& \ x \geq 0 \ \wedge \ t \leq 1\};$$

$$\text{if}(x=0) \ v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv)^*](0 \leq x \leq 5)$$

Proof?



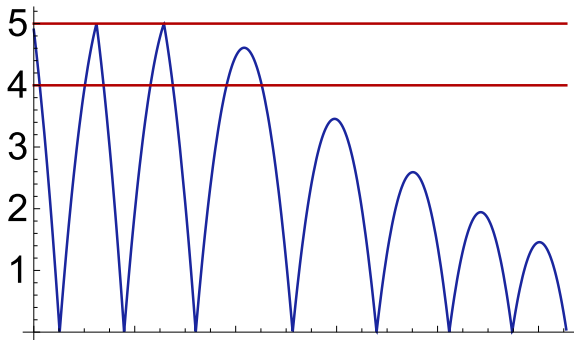
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\};$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv)^*](0 \leq x \leq 5)$$

Proof?



Conjecture (Quantum can play ping-pong safely)

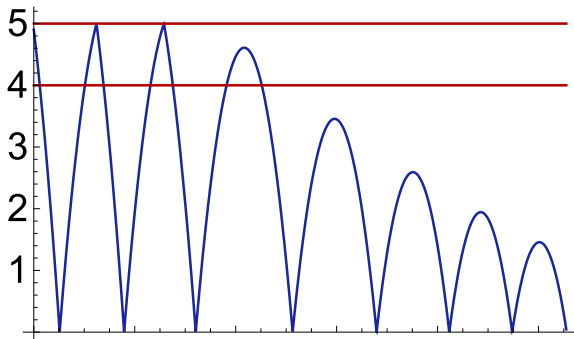
$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\};$$

$$\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv)^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes



Wind up a clock

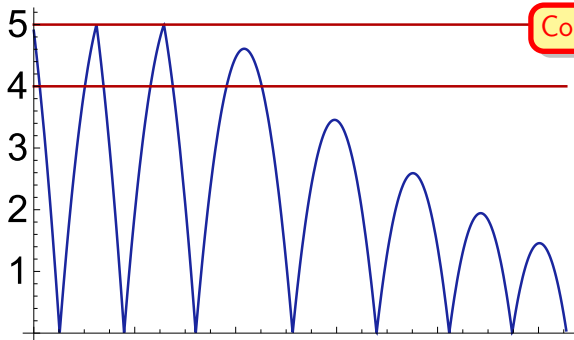
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\text{if}(x=0) \ v := -cv \ \text{else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \ \& \ x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes

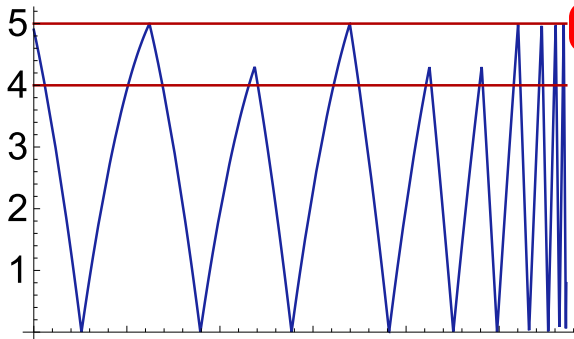


Control action before physics

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\text{if}(x=0) v := -cv \text{ else if}(4 \leq x \leq 5 \wedge v \geq 0) v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes



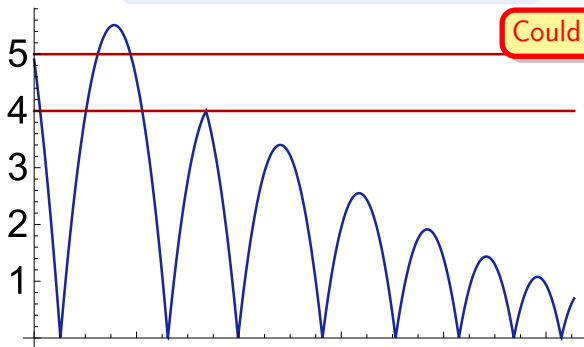
Could act early or late

Quantum the Time-triggered Ping-Pong Ball

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\text{if}(x=0) \ v := -cv \ \text{else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



Could miss event off control cycle

Delays vs. Events

- 1 Periodically/frequently monitor for an event with a polling frequency / reaction time.
- 2 Delays may make the controller miss events.
- 3 Discrepancy between event-triggered idea vs. real time-triggered implementation.
- 4 Issues indicate poor event abstraction.
- 5 Slow controllers monitoring small regions of a fast moving system.
- 6 Controller needs to be aware of its own delay.

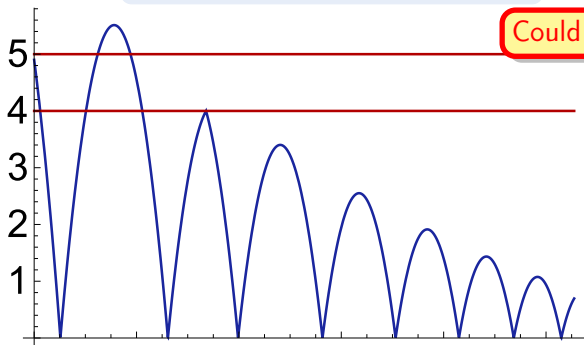
Outwit the Cartesian Demon

Skeptical about the truth of all beliefs
until justification has been found.

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g > 0 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\text{if}(x=0) \ v := -cv \ \text{else if}(4 \leq x \leq 5 \wedge v \geq 0) \ v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



Could miss event off control cycle

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

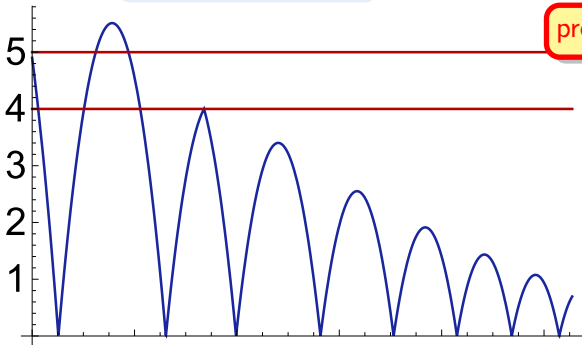
$$[(\text{if}(x=0) v := -cv \text{ else if}(x > 5\frac{1}{2} - v \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes

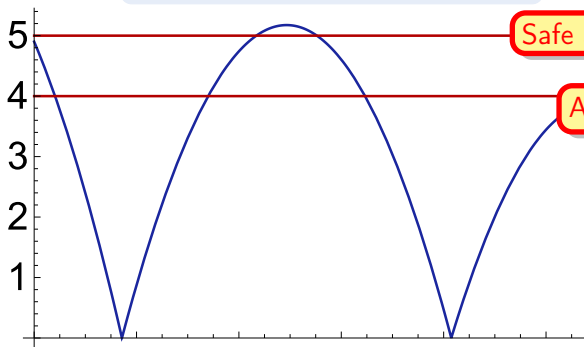
predict 1s: $x + v - \frac{g}{2} > 5$



Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\text{if}(x=0) v := -cv \text{ else if}(x > 5\frac{1}{2} - v \wedge v \geq 0) v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



Safe after 1 s but not until then

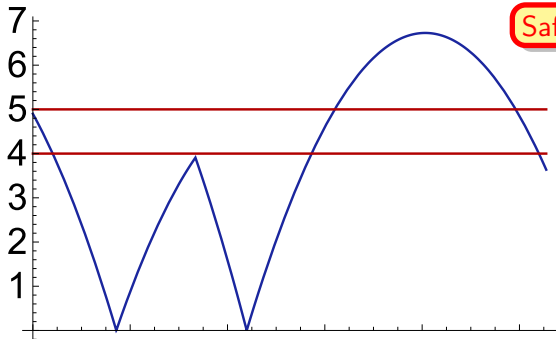
All depends on sampling

Quantum the Time-triggered Ping-Pong Ball

Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$
$$[(\text{if}(x=0) v := -cv \text{ else if}(x > 5\frac{1}{2} - v \wedge v \geq 0) v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



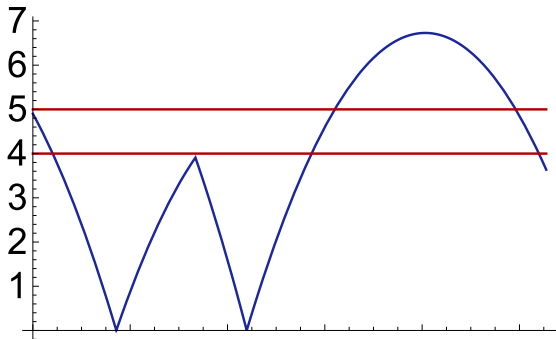
Safe after 1 s but not until then

All depends on sampling

Design-by-Invariant

$$2gx = 2gH - v^2 \wedge x \geq 0 \wedge c = 1 \wedge g > 0$$

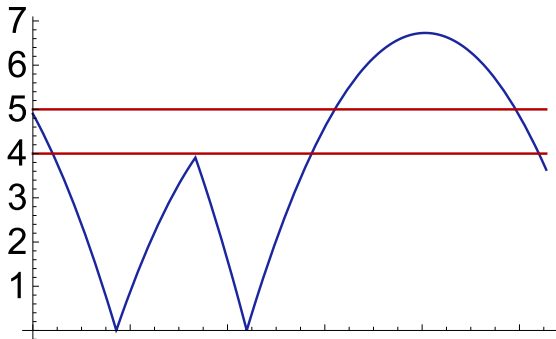
bouncing ball invariant



Design-by-Invariant

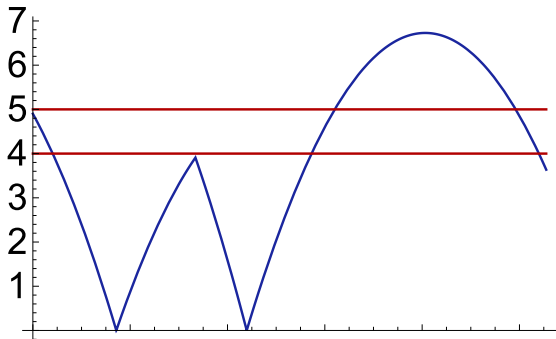
$$2gx = 2gH - v^2 \wedge x \geq 0 \wedge c = 1 \wedge g = 1$$

simplify arithmetic



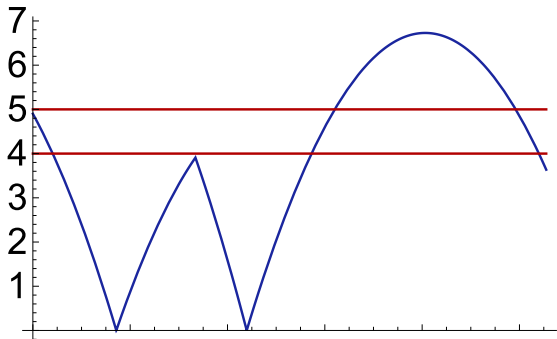
Design-by-Invariant

$$2x = 2H - v^2 \wedge x \geq 0$$



Design-by-Invariant

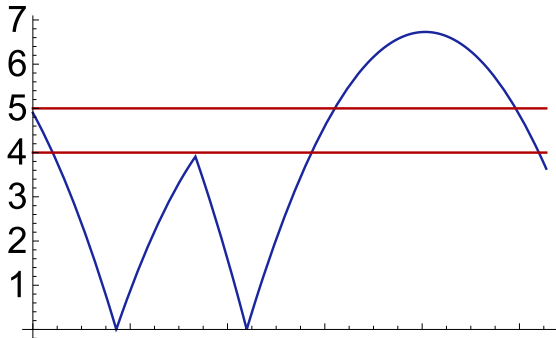
$$2x = 2 \cdot H - v^2 \wedge x \geq 0$$



Design-by-Invariant

$$2x = 2 \cdot 5 - v^2 \wedge x \geq 0$$

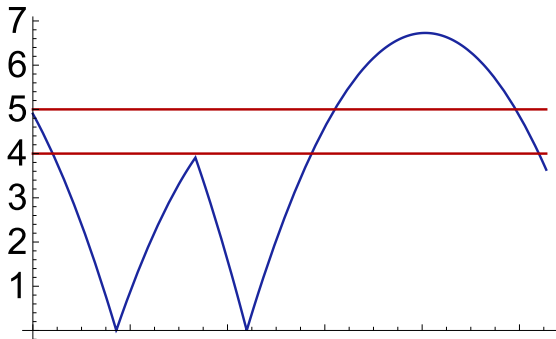
critical height



Design-by-Invariant

$$2x > 2 \cdot 5 - v^2 \wedge x \geq 0$$

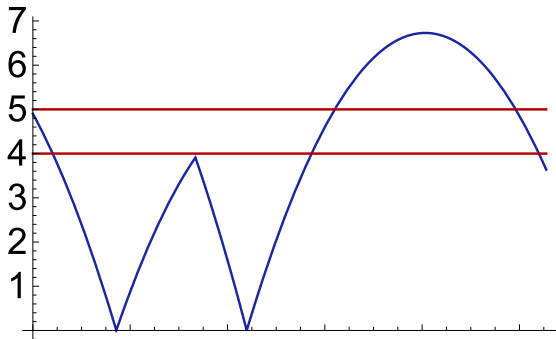
potential exceeds safe height



Design-by-Invariant

$$2x > 2 \cdot 5 - v^2 \wedge x \geq 0$$

use invariant for control



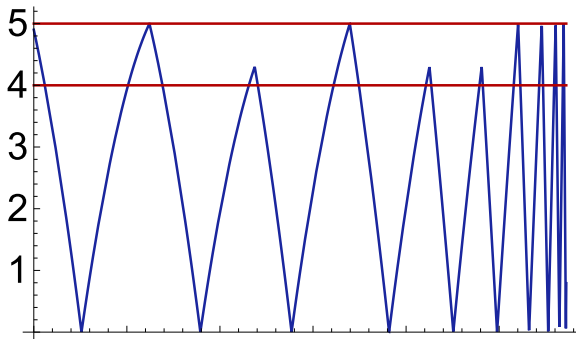
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 \geq c \geq 0 \wedge f \geq 0 \rightarrow$$

$$[(\text{if}(x=0) \ v := -cv \ \text{else if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) \ v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \ \& \ x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes



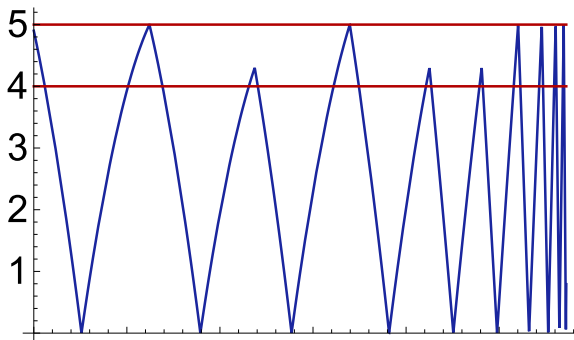
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge \mathbf{1} = \mathbf{c} \wedge \mathbf{f} = \mathbf{1} \rightarrow$$

$$[(\text{if}(x=0) \ v := -cv \ \text{else if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) \ v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \ \& \ x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes



Just for simplicity

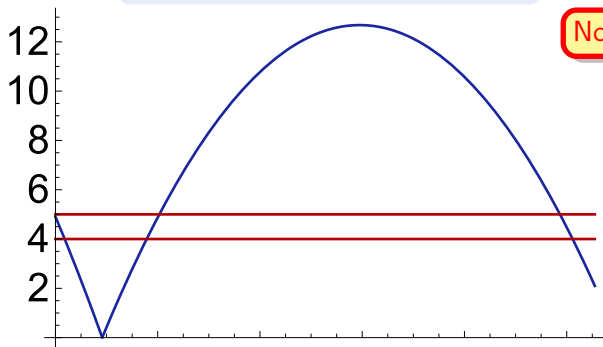
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge f = 1 \rightarrow$$

$$[(\text{if}(x=0) v := -cv \text{ else if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says no!



No control near ground

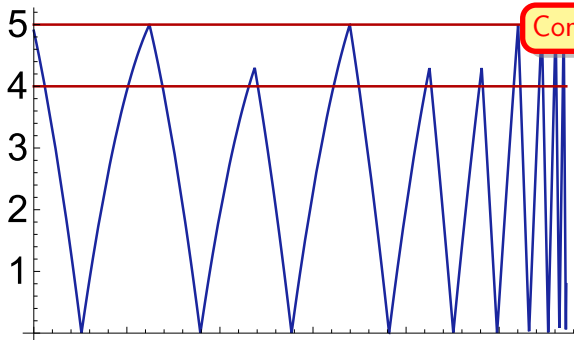
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes



Control despite ground

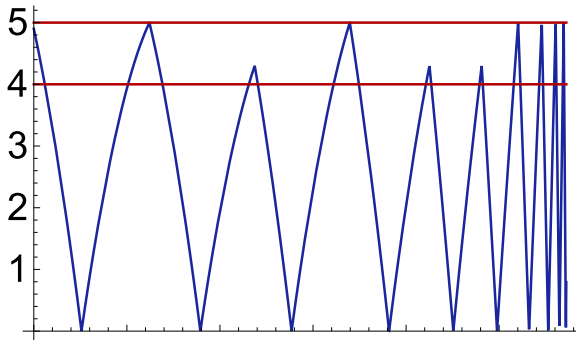
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says yes



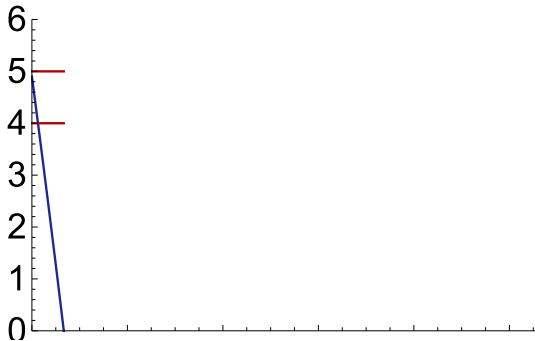
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says yes but should have said no!

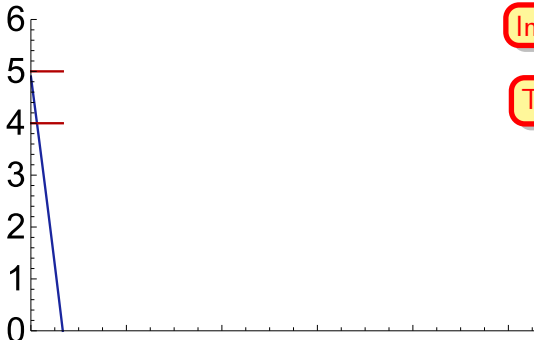


Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$
$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2) \wedge v \geq 0) v := -fv;$$
$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof?

Ask René Descartes who says yes but should have said no!



Invariants are invariants!

True ever \rightsquigarrow true always

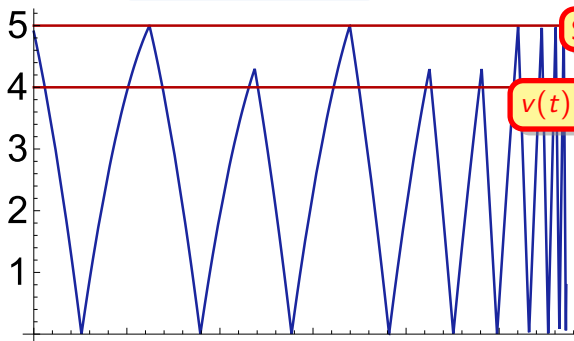
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2 \wedge v < 1) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes



Slow turnaround

$$v(t) = v - gt = v - t < 0$$

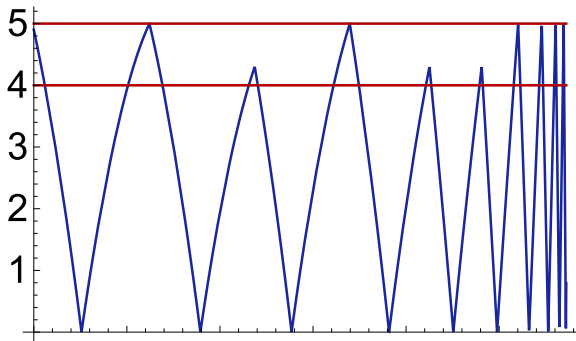
Conjecture (Quantum can play ping-pong safely)

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g = 1 \wedge 1 = c \wedge 1 = f \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2 \wedge v < 1) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof? Ask René Descartes who says yes



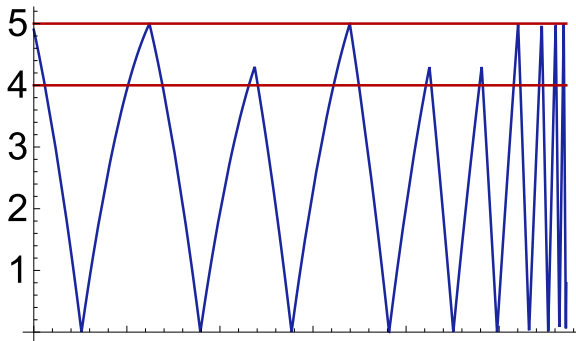
Proposition (▶) Quantum can play ping-pong safely in real-time

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g=1 > 0 \wedge 1=c \geq 0 \wedge 1=f \geq 0 \rightarrow$$

$$\left[\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2 \wedge v < 1) \wedge v \geq 0) v := -fv; \right.$$

$$\left. t := 0; \{x' = v, v' = -g, t' = 1 \ \& \ x \geq 0 \wedge t \leq 1\}^* \right] (0 \leq x \leq 5)$$

Proof



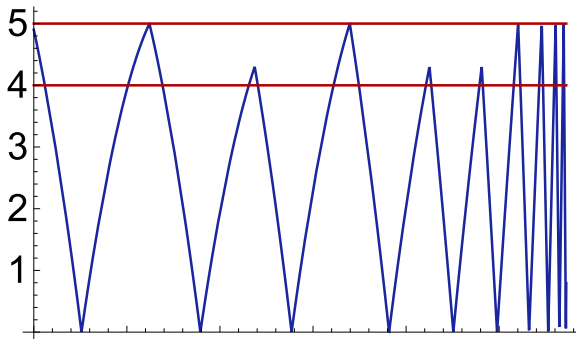
Proposition (▶) Quantum can play ping-pong safely in real-time

$$0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g=1 > 0 \wedge 1=c \geq 0 \wedge 1=f \geq 0 \rightarrow$$

$$\left[\left(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2 \wedge v < 1) \wedge v \geq 0) v := -fv; \right. \right.$$

$$\left. \left. t := 0; \{x' = v, v' = -g, t' = 1 \ \& \ x \geq 0 \wedge t \leq 1\}^* \right] (0 \leq x \leq 5)$$

Proof @invariant($2x = 2H - v^2 \wedge x \geq 0 \wedge x \leq 5$)



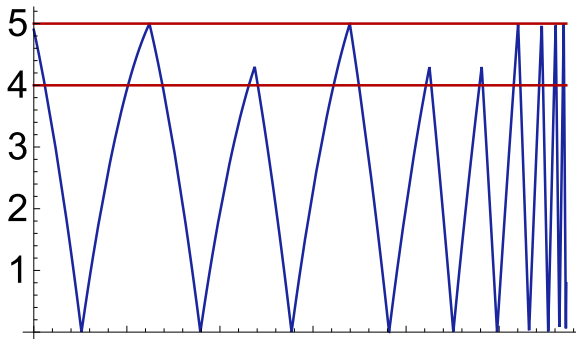
Proposition (▶) Quantum can play ping-pong safely in real-time

$$2x = 2H - v^2 \wedge 0 \leq x \wedge x \leq 5 \wedge v \leq 0 \wedge g=1 > 0 \wedge 1=c \geq 0 \wedge 1=f \geq 0 \rightarrow$$

$$[(\text{if}(x=0) v := -cv; \text{if}((x > 5\frac{1}{2} - v \vee 2x > 2.5 - v^2 \wedge v < 1) \wedge v \geq 0) v := -fv;$$

$$t := 0; \{x' = v, v' = -g, t' = 1 \& x \geq 0 \wedge t \leq 1\})^*](0 \leq x \leq 5)$$

Proof @invariant($2x = 2H - v^2 \wedge x \geq 0 \wedge x \leq 5$)



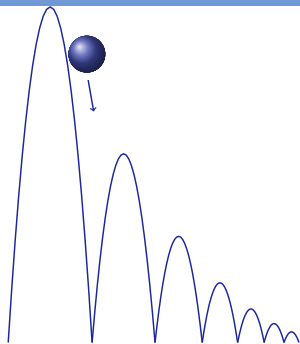


- 1 Learning Objectives
- 2 Delays in Control
 - The Impact of Delays on Event Detection
 - Cartesian Demon
 - Model-Predictive Control Basics
 - Design-by-Invariant
 - Controlling the Control Points
 - Sequencing and Prioritizing Reactions
 - Time-Triggered Verification
- 3 Summary

- 1 Common paradigm for designing real controllers
- 2 Periodical or pseudo-periodical control (jitter)
- 3 Expects delays, expects inertia
- 4 Implementation: discrete-time sensing
- 5 Predict events, not just: $\text{if}(\text{eventnow}(x)) \dots$
- 6 Safe controllers know their own reaction delays
- 7 Burden of event detection brought to attention of CPS programmer
- 8 Time-triggered controls are implementable and more robust, but make design and verification more challenging!
- 9 Use knowledge gained from verified event-triggered model as a basis for designing a time-triggered controller



- 4 Appendix
 - Zeno's Quantum Turtles

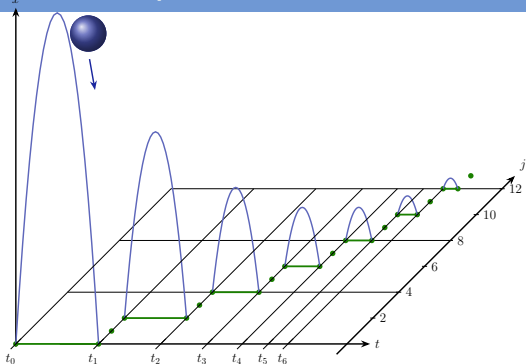


Example (Quantum the Bouncing Ball)

$$\begin{aligned} &(\{x' = v, v' = -g \ \& \ x \geq 0\}; \\ &\text{if}(x = 0) \ v := -cv)^* \end{aligned}$$



How Quantum Met Achilles and His Tortoise

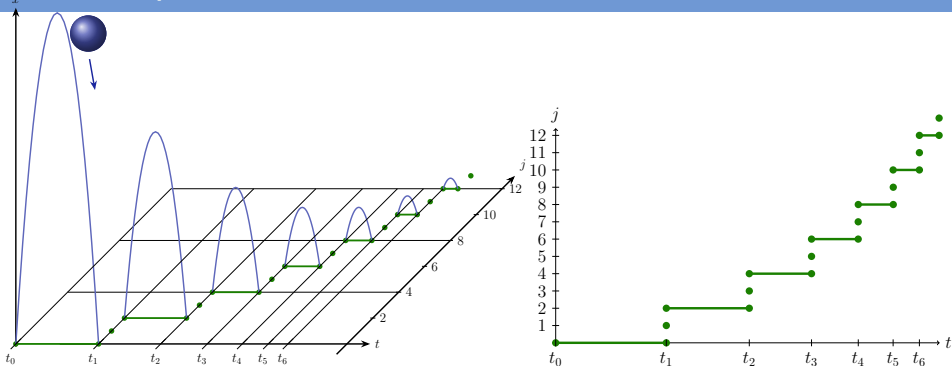


Example (Quantum the Bouncing Ball)

$$\left(\{x' = v, v' = -g \ \& \ x \geq 0\};\right. \\ \left.\text{if}(x = 0) \ v := -cv\right)^*$$



How Quantum Met Achilles and His Tortoise



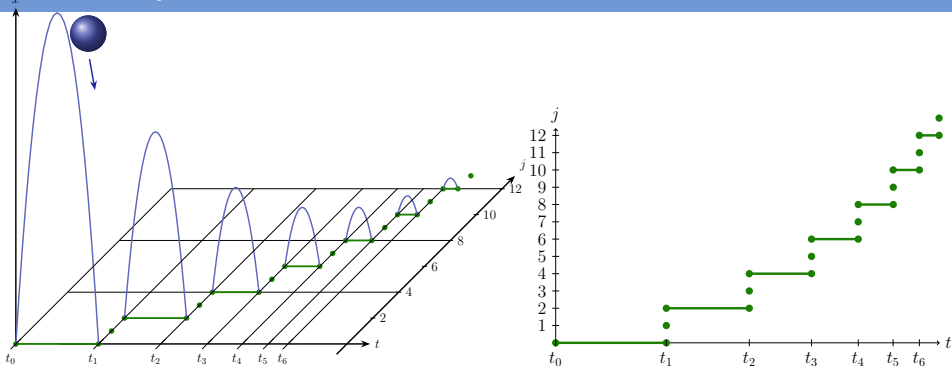
Example (Quantum the Bouncing Ball)

$$\{x' = v, v' = -g \ \& \ x \geq 0\};$$

$$\text{if}(x = 0) \ v := -cv)^*$$



How Quantum Met Achilles and His Tortoise

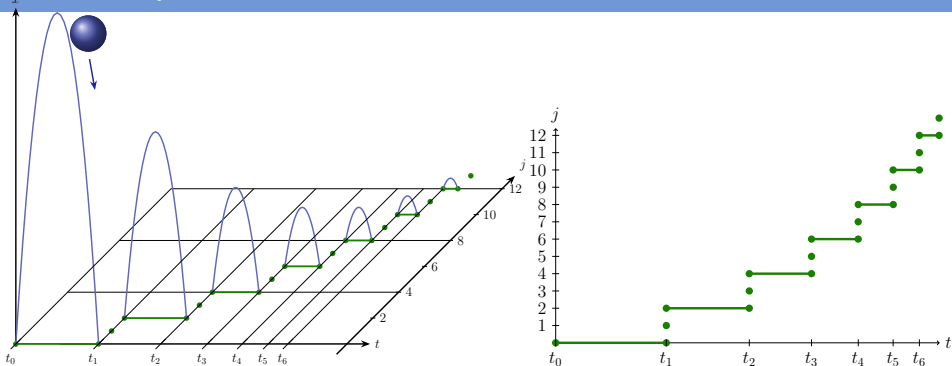


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$$



How Quantum Met Achilles and His Tortoise

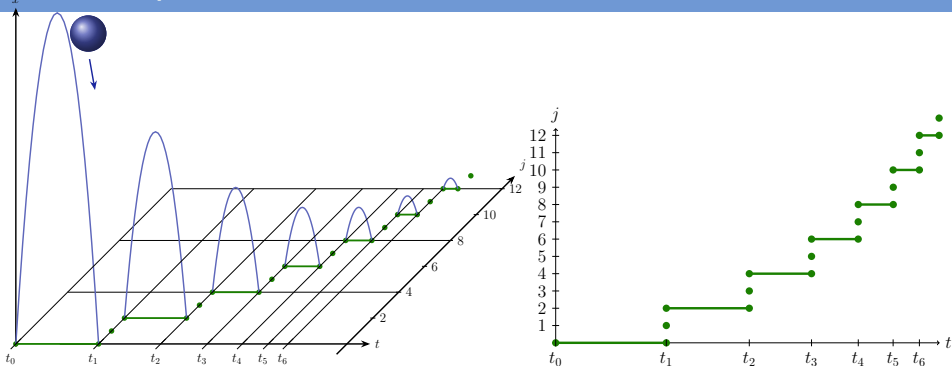


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i}$$



How Quantum Met Achilles and His Tortoise

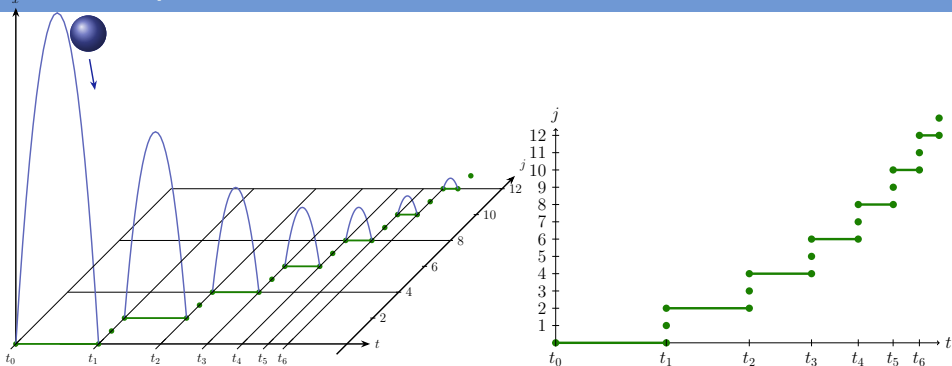


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}}$$



How Quantum Met Achilles and His Tortoise

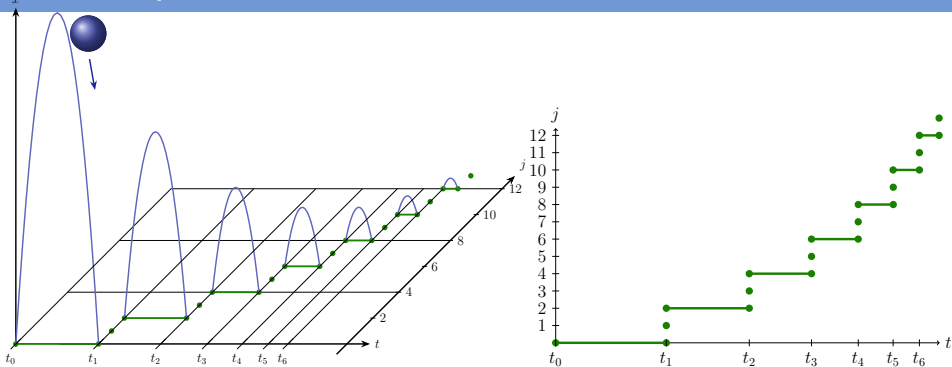


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2$$



How Quantum Met Achilles and His Tortoise

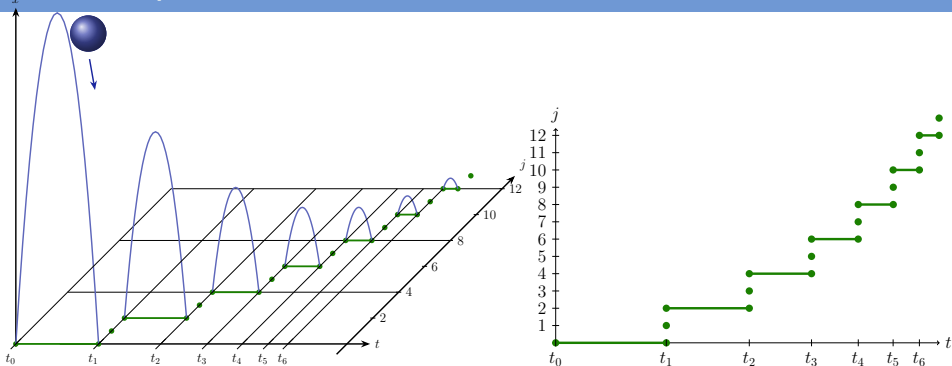


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



How Quantum Met Achilles and His Tortoise

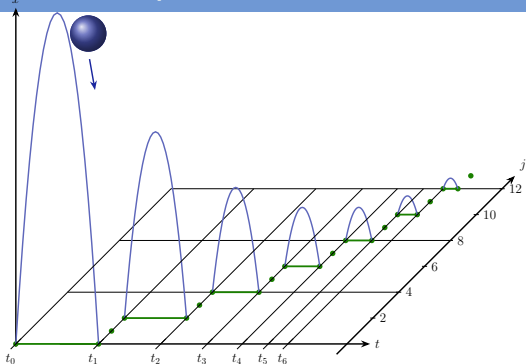


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



How Quantum Met Achilles and His Tortoise

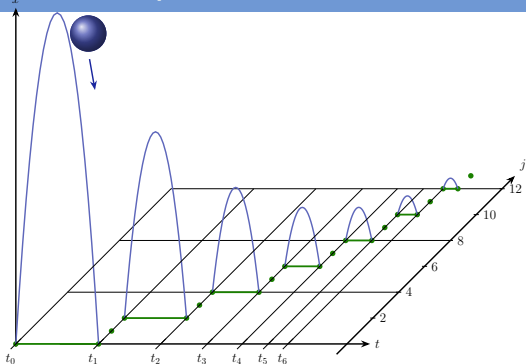


Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



How Quantum Met Achilles and His Tortoise



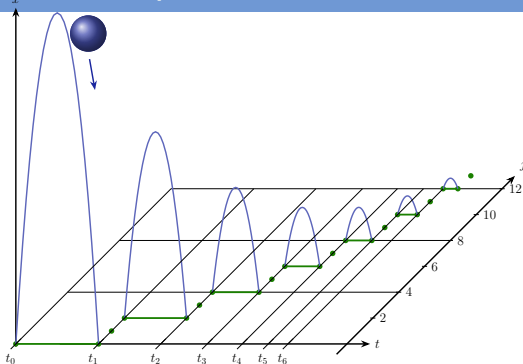
I don't exist

Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



How Quantum Met Achilles and His Tortoise



I don't exist

Example (Quantum the Bouncing Ball experiences time)

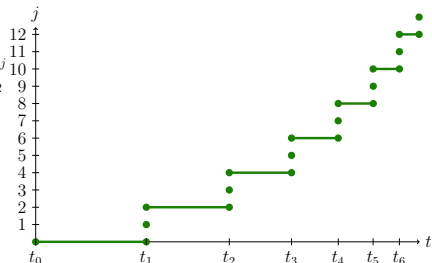
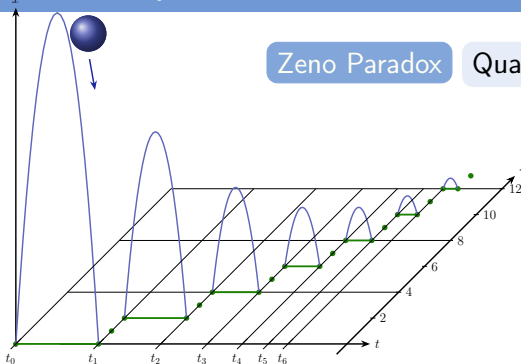
$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



How Quantum Met Achilles and His Tortoise

Zeno Paradox

Quantum's model causes a time freeze



Example (Quantum the Bouncing Ball experiences time)

$$1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots = \sum_{i=0}^{\infty} \frac{1}{2^i} = \frac{1}{1 - \frac{1}{2}} = 2 < \infty$$



André Platzer.

Logical Foundations of Cyber-Physical Systems.

Springer, Switzerland, 2018.

URL: <http://www.springer.com/978-3-319-63587-3>,

doi:10.1007/978-3-319-63588-0.



André Platzer.

Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics.

Springer, Heidelberg, 2010.

doi:10.1007/978-3-642-14509-4.



Sarah M. Loos and André Platzer.

Differential refinement logic.

In Martin Grohe, Eric Koskinen, and Natarajan Shankar, editors, *LICS*, pages 505–514, New York, 2016. ACM.

doi:10.1145/2933575.2934555.