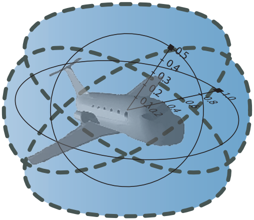
Logical Foundations of Cyber-Physical Systems

André Platzer

aplatzer@cs.cmu.edu
Computer Science Department
Carnegie Mellon University, Pittsburgh, PA

http://symbolaris.com/



Outline

1. CPS are Multi-Dynamical Systems
   - Hybrid Systems
   - Hybrid Games
   - Stochastic Hybrid Systems
   - Distributed Hybrid Systems
2. Dynamic Logic of Multi-Dynamical Systems
3. Proofs for CPS
4. Theory of CPS
   - Soundness and Completeness
   - Differential Invariants
5. Applications
6. Summary

Can you trust a computer to control physics?

Can you trust a computer to control physics?

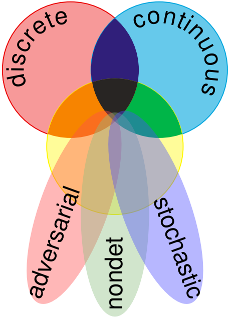
Rationale

1. Safety guarantees require analytic foundations
2. Foundations revolutionized digital computer science & society
3. Need even stronger foundations when software reaches out into our physical world

CPS are Multi-Dynamical Systems

CPS Dynamics Bee

CPS are characterized by multiple facets of dynamical systems.



CPS Compositions

CPS combine many simple dynamical effects.

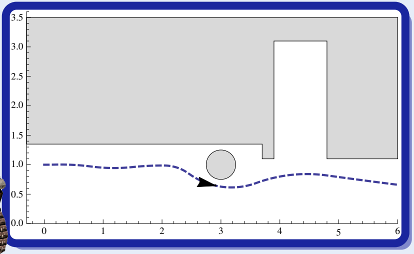
Tame Parts

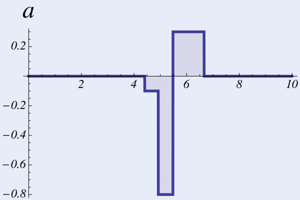
Exploiting compositionality tames complexity.

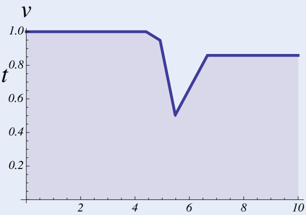
Challenge (Hybrid Systems)

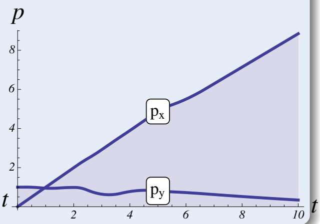
Fixed rule describing state evolution with both

- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)





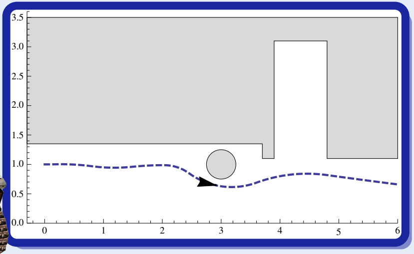


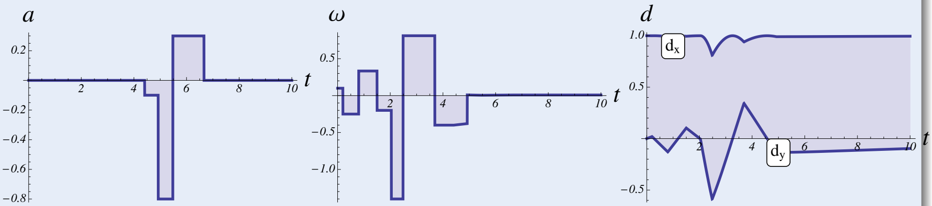


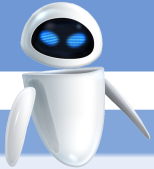
Challenge (Hybrid Systems)

Fixed rule describing state evolution with both

- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)



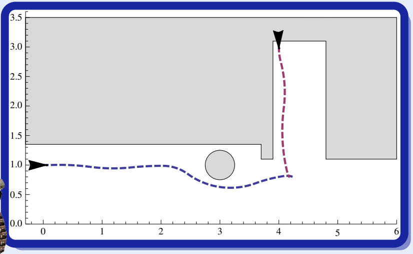


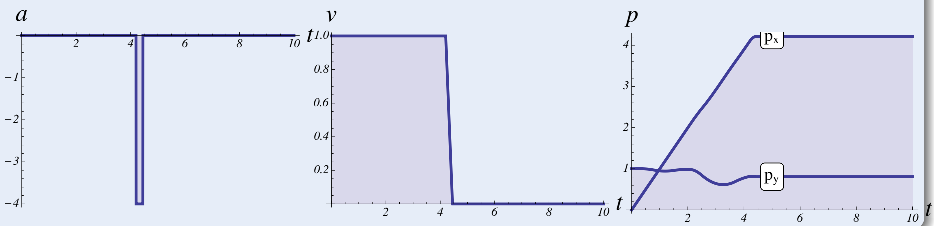


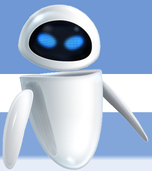
Challenge (Hybrid Systems)

Fixed rule describing state evolution with both

- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)



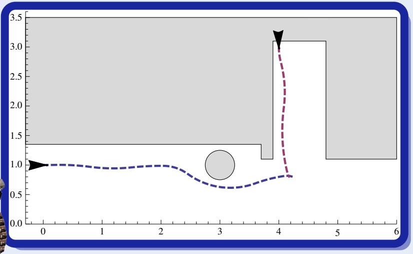


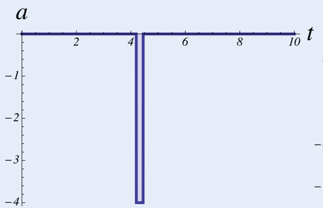


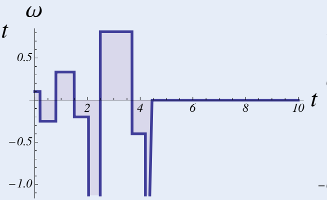
Challenge (Hybrid Systems)

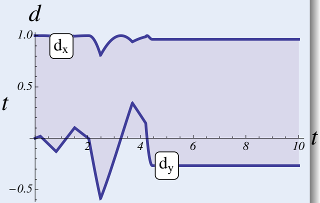
Fixed rule describing state evolution with both

- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)

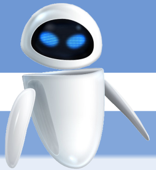








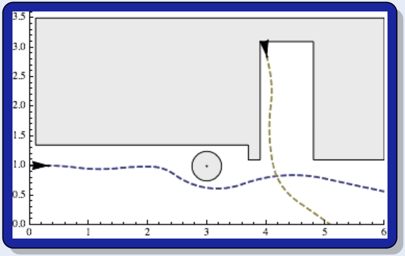
CPS Analysis: Other Agents

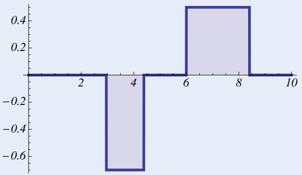


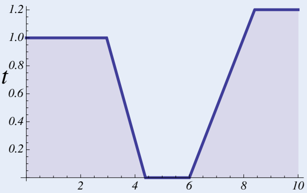
Challenge (Hybrid Games)

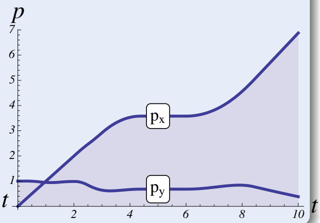
Game rules describing play evolution with

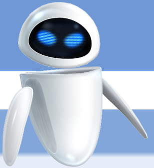
- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)
- Adversarial dynamics (Angel ◇ vs. Demon □)







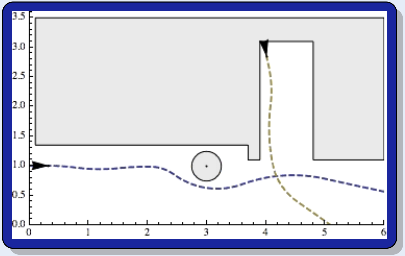


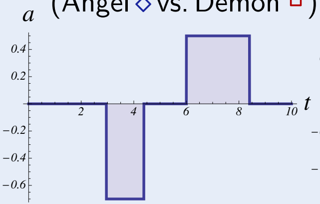


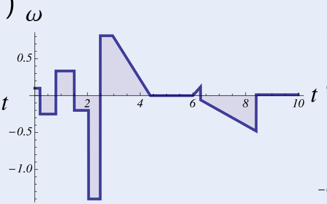
Challenge (Hybrid Games)

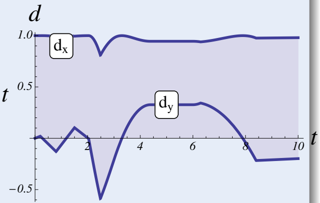
Game rules describing play evolution with

- Discrete dynamics (control decisions)
- Continuous dynamics (differential equations)
- Adversarial dynamics (Angel ◇ vs. Demon □)





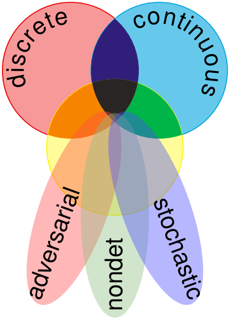




CPS are Multi-Dynamical Systems

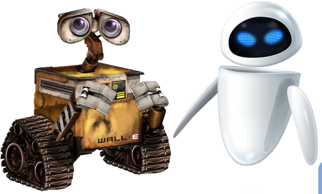
hybrid systems

HS = discrete + ODE



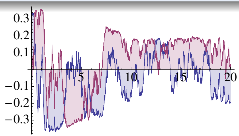
hybrid games

HG = HS + adversary



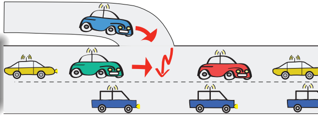
stochastic hybrid sys.

SHS = HS + stochastics

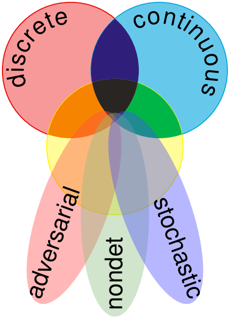


distributed hybrid sys.

DHS = HS + distributed



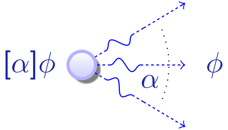
Family of Differential Dynamic Logics

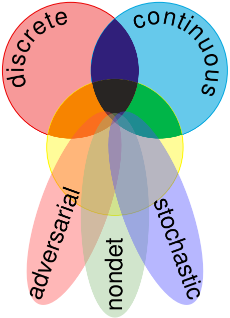


Family of Differential Dynamic Logics

differential dynamic logic

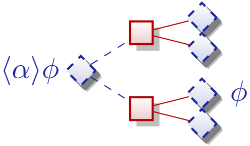
$d\mathcal{L} = \mathrm{DL} + \mathrm{HP}$





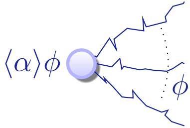
differential game logic

$dG\mathcal{L} = \mathrm{GL} + \mathrm{HG}$



stochastic differential DL

$Sd\mathcal{L} = \mathrm{DL} + \mathrm{SHP}$



quantified differential DL

$Qd\mathcal{L} = \mathrm{FOL} + \mathrm{DL} + \mathrm{QHP}$

Differential Dynamic Logic: Axiomatization

equations of truth

[:=] $[x := \theta]\phi(x) \leftrightarrow \phi(\theta)$

[?] $[?H]\phi \leftrightarrow (H \rightarrow \phi)$

['] $[x' = f(x)]\phi \leftrightarrow \forall t{\geq}0\, [x := y(t)]\phi$ $\qquad (y'(t) = f(y))$

[∪] $[\alpha \cup \beta]\phi \leftrightarrow [\alpha]\phi \land [\beta]\phi$

[;] $[\alpha; \beta]\phi \leftrightarrow [\alpha][\beta]\phi$

[*] $[\alpha^*]\phi \leftrightarrow \phi \land [\alpha][\alpha^*]\phi$

K $[\alpha](\phi \rightarrow \psi) \rightarrow ([\alpha]\phi \rightarrow [\alpha]\psi)$

I $[\alpha^*](\phi \rightarrow [\alpha]\phi) \rightarrow (\phi \rightarrow [\alpha^*]\phi)$

C $[\alpha^*]\forall v{>}0\, (\varphi(v) \rightarrow \langle\alpha\rangle\varphi(v-1)) \rightarrow \forall v\, (\varphi(v) \rightarrow \langle\alpha^*\rangle\exists v{\leq}0\, \varphi(v))$

LICS'12

Complete Proof Theory of Hybrid Systems

Theorem (Sound & Complete) (J.Autom.Reas. 2008, LICS'12)

d$\mathcal{L}$ *calculus is a sound & complete axiomatization of hybrid systems relative to differential equations* **or** *discrete dynamics.*

Proof 25pp

Corollary (Complete Proof-theoretical Alignment & Bridging)

proving continuous = proving hybrid = proving discrete

Complete Proof Theory of Hybrid Systems

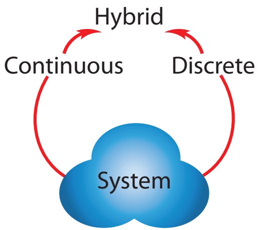
Theorem (Sound & Complete) (J.Autom.Reas. 2008, LICS'12)

d$\mathcal{L}$ *calculus is a sound & complete axiomatization of hybrid systems relative to differential equations* **or** *discrete dynamics.*

Proof 25pp

Corollary (Complete Proof-theoretical Alignment & Bridging)

proving continuous = proving hybrid = proving discrete



Complete Proof Theory of Hybrid Systems

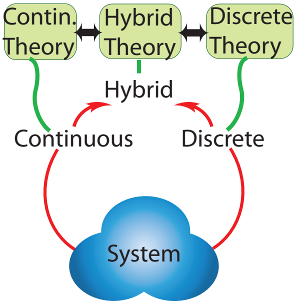
Theorem (Sound & Complete) (J.Autom.Reas. 2008, LICS'12)

d$\mathcal{L}$ *calculus is a sound & complete axiomatization of hybrid systems relative to differential equations* **or** *discrete dynamics.*

Proof 25pp

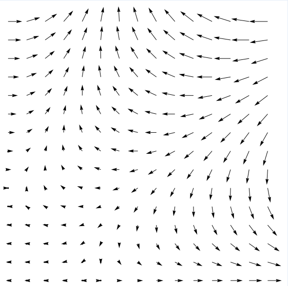
Corollary (Complete Proof-theoretical Alignment & Bridging)

proving continuous = proving hybrid = proving discrete

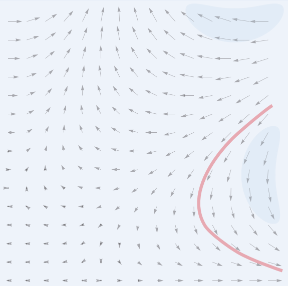


Differential Invariants for Differential Equations

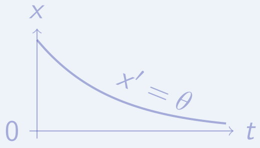
Differential Invariant

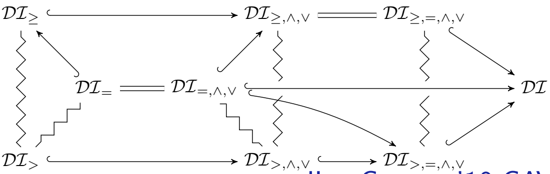


Differential Cut



Differential Ghost





Logic

Provability study

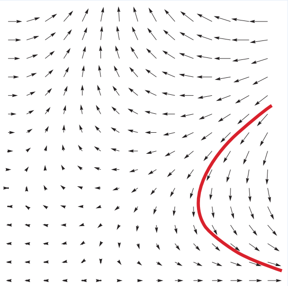
Math

Character-istic PDE

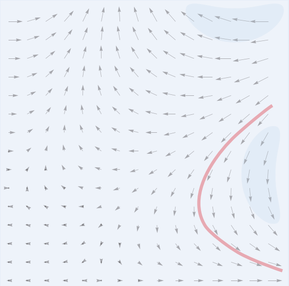
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

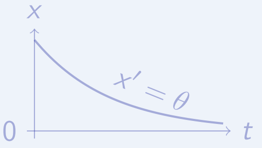
Differential Invariant

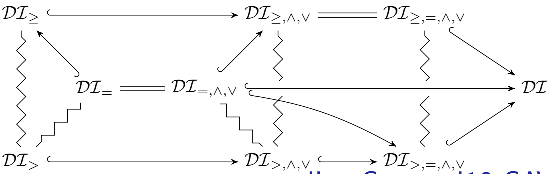


Differential Cut



Differential Ghost





Logic

Provability study

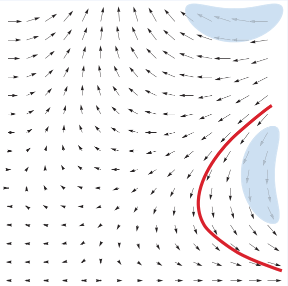
Math

Character-istic PDE

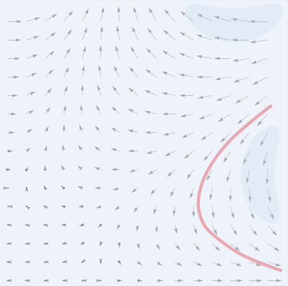
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

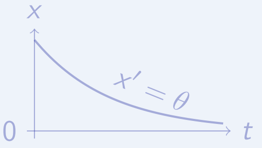
Differential Invariant

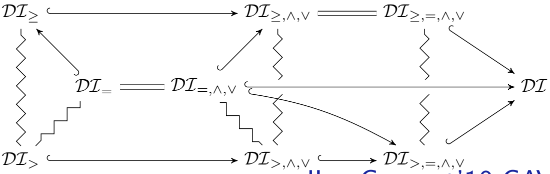


Differential Cut



Differential Ghost





Logic

Provability study

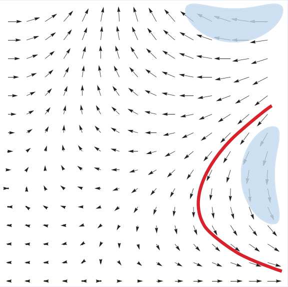
Math

Character-istic PDE

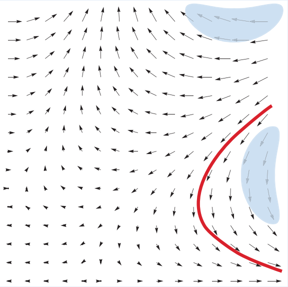
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

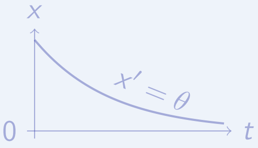
Differential Invariant

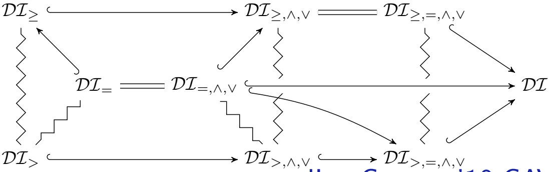


Differential Cut



Differential Ghost





Logic

Provability study

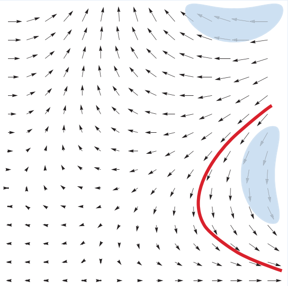
Math

Character-istic PDE

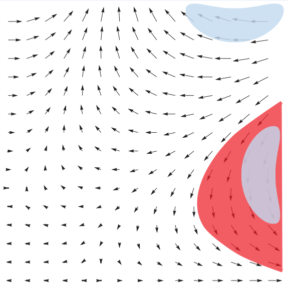
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

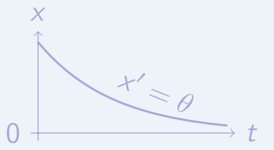
Differential Invariant

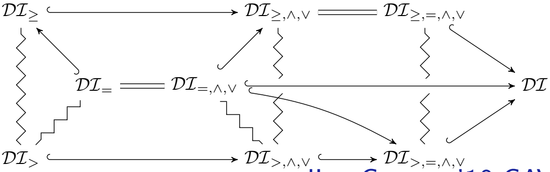


Differential Cut



Differential Ghost





Logic

Provability study

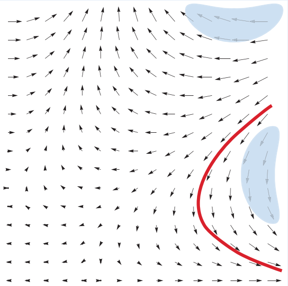
Math

Character-istic PDE

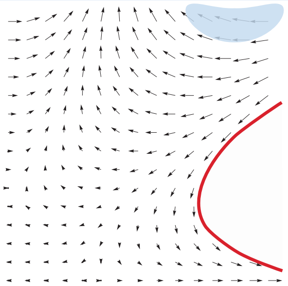
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

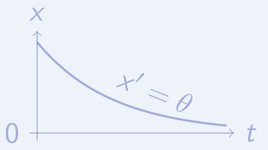
Differential Invariant

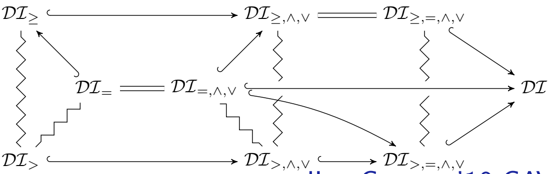


Differential Cut



Differential Ghost





Logic

Provability study

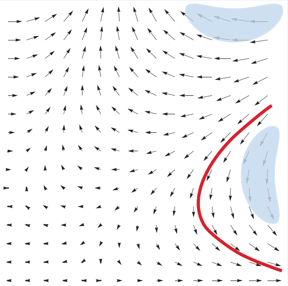
Math

Characteristic PDE

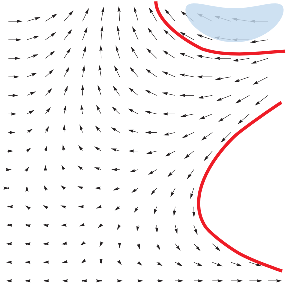
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

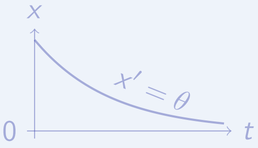
Differential Invariant

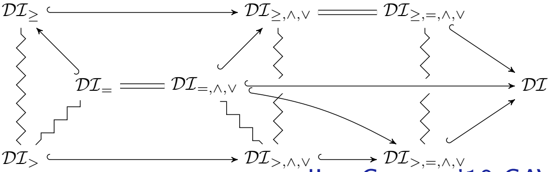


Differential Cut



Differential Ghost





Logic

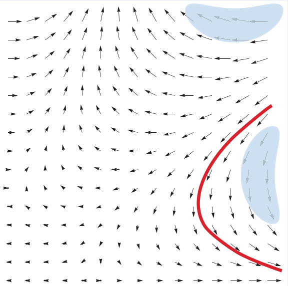
Provability study

Math

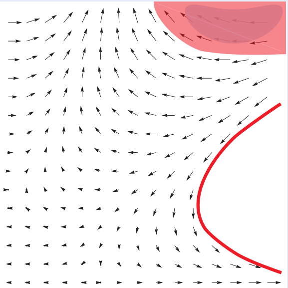
Character-istic PDE

JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

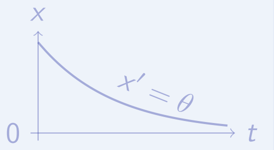
Differential Invariant

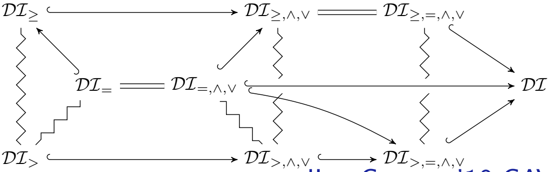


Differential Cut



Differential Ghost





Logic

Provability study

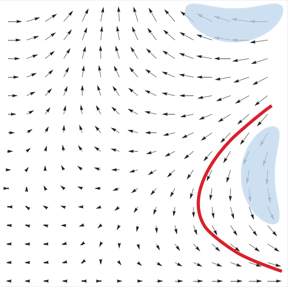
Math

Character-istic PDE

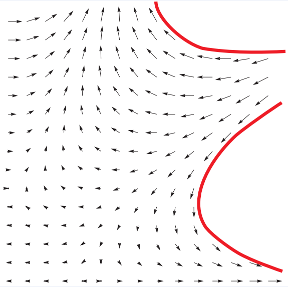
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

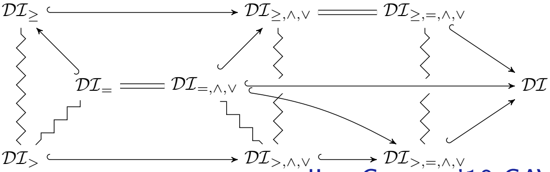
Differential Invariant



Differential Cut



Differential Ghost



Logic

Provability study

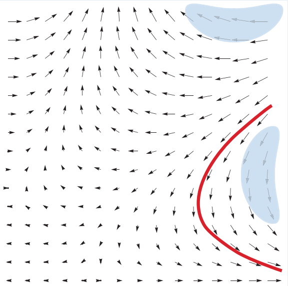
Math

Characteristic PDE

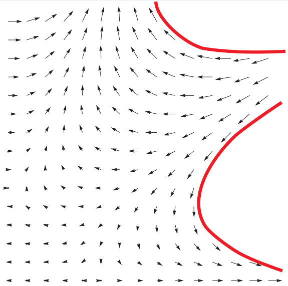
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

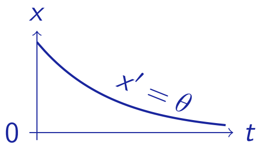
Differential Invariant

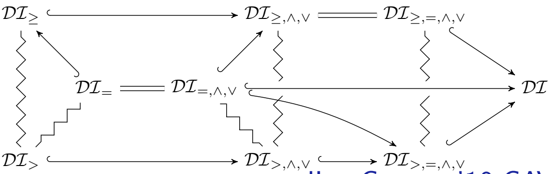


Differential Cut



Differential Ghost





Logic

Provability study

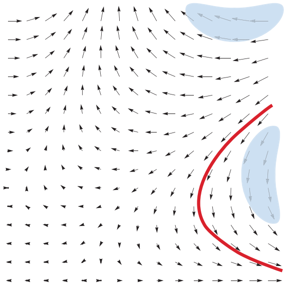
Math

Character-istic PDE

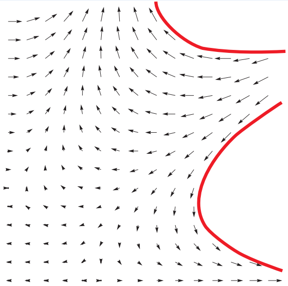
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

Differential Invariants for Differential Equations

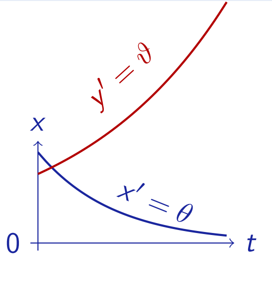
Differential Invariant

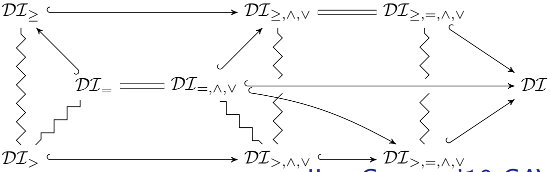


Differential Cut



Differential Ghost





Logic

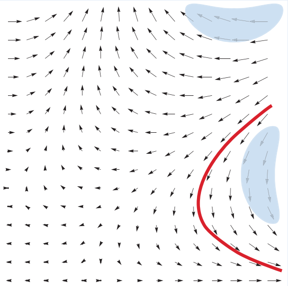
Provability study

Math

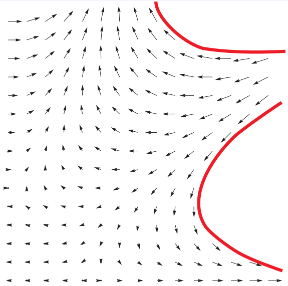
Character-istic PDE

JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

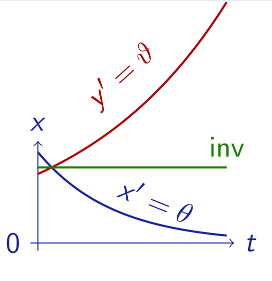
Differential Invariant

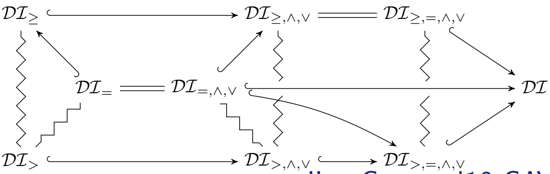


Differential Cut



Differential Ghost





Logic

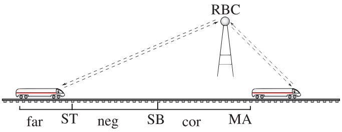
Provability study

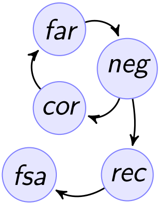
Math

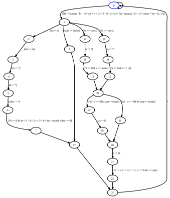
Character-istic PDE

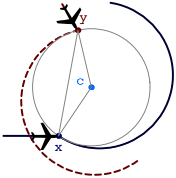
JLogComput'10,CAV'08,FMSD'09,LMCS'12,ITP'12

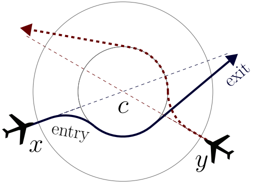
Successful CPS Proofs

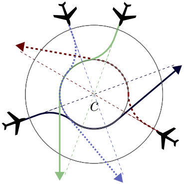


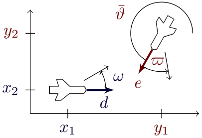


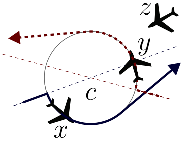


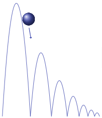


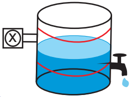






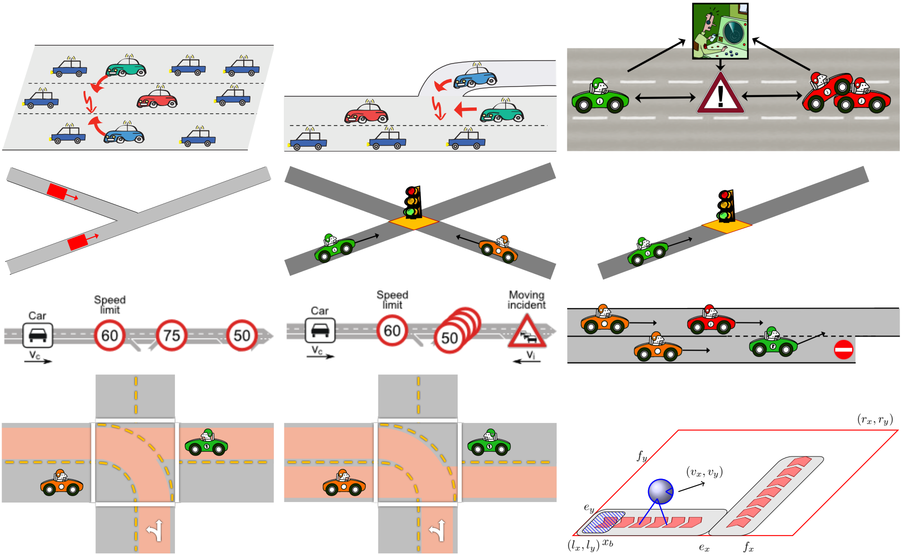






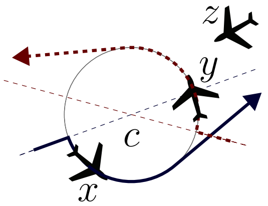
ICFEM'09,CAV'08,FM'09,HSCC'11

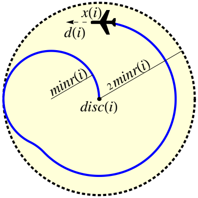
Successful CPS Proofs

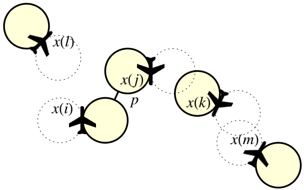


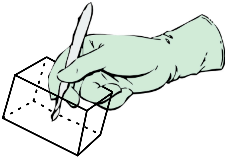
FM'11,LMCS'12,ICCPS'12,ITSC'11,IJCAR'12

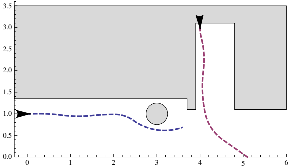
Successful CPS Proofs

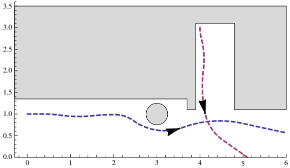


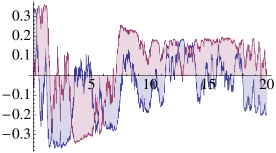


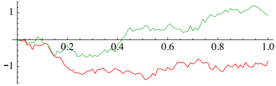


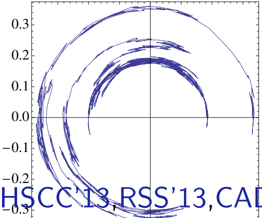










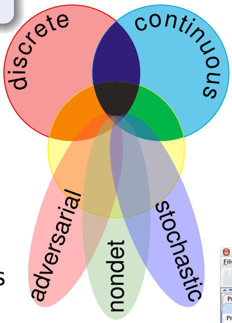


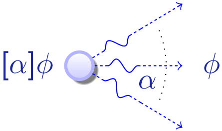
HSCC'11,HSCC'13,HSCC'13,RSS'13,CADE'12

Logical Foundations of Cyber-Physical Systems

differential dynamic logic

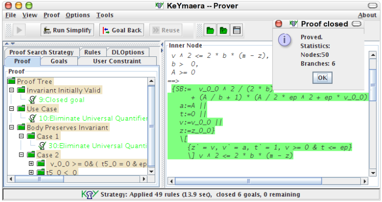
$d\mathcal{L} = DL + HP$

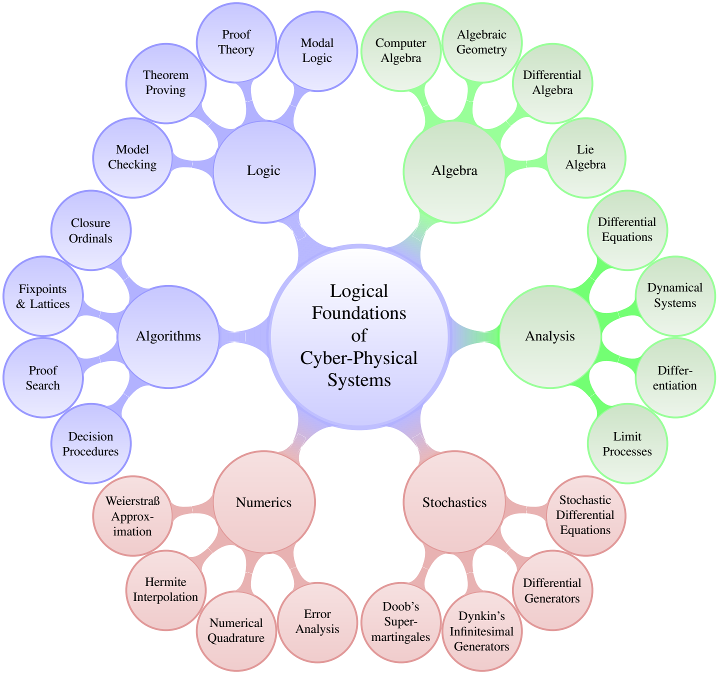




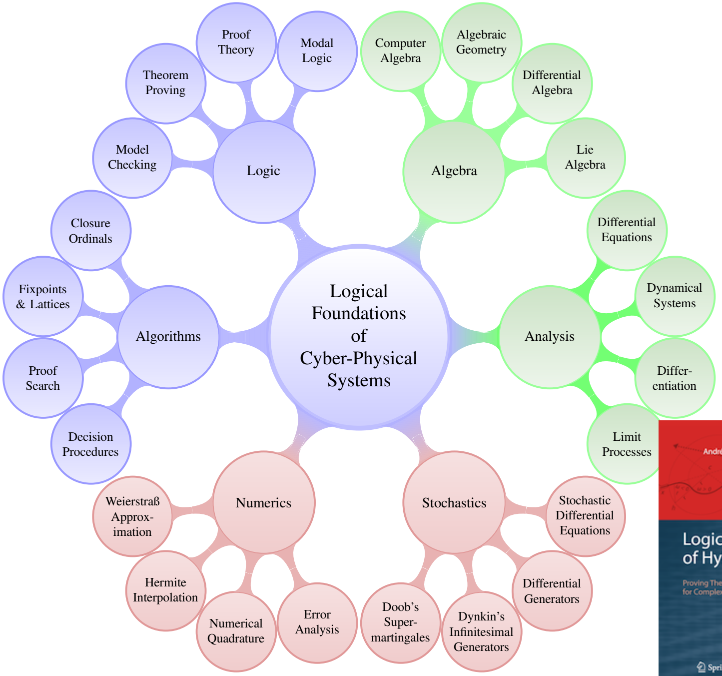
- Multi-dynamical systems
- Combine simple dynamics
- Tame complexity
- Logic & proofs for CPS
- Theory of CPS
- Applications

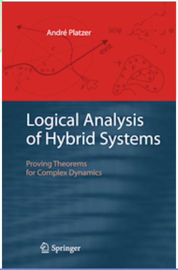
KeYmaera





Logical Foundations of Cyber-Physical Systems
Logic
Proof Theory
Modal Logic
Theorem Proving
Model Checking
Algebra
Computer Algebra
Algebraic Geometry
Differential Algebra
Lie Algebra
Analysis
Differential Equations
Dynamical Systems
Differ- entiation
Limit Processes
Stochastics
Stochastic Differential Equations
Differential Generators
Dynkin's Infinitesimal Generators
Doob's Super- martingales
Numerics
Error Analysis
Numerical Quadrature
Hermite Interpolation
Weierstraß Approx- imation
Algorithms
Closure Ordinals
Fixpoints & Lattices
Proof Search
Decision Procedures


Logical Foundations of Cyber-Physical Systems
Logic
Proof Theory
Modal Logic
Theorem Proving
Model Checking
Algebra
Computer Algebra
Algebraic Geometry
Differential Algebra
Lie Algebra
Algorithms
Closure Ordinals
Fixpoints & Lattices
Proof Search
Decision Procedures
Analysis
Differential Equations
Dynamical Systems
Differ-entiation
Limit Processes
Numerics
Weierstraß Approx-imation
Hermite Interpolation
Numerical Quadrature
Error Analysis
Stochastics
Stochastic Differential Equations
Differential Generators
Doob's Super-martingales
Dynkin's Infinitesimal Generators


André Platzer
Logical Analysis of Hybrid Systems
Proving Theorems for Complex Dynamics
Springer

André Platzer.
Logics of dynamical systems.
In *LICS* [9], pages 13–24.
doi:10.1109/LICS.2012.13.

André Platzer.
Differential dynamic logic for hybrid systems.
J. Autom. Reas., 41(2):143–189, 2008.
doi:10.1007/s10817-008-9103-8.

André Platzer.
The complete proof theory of hybrid systems.
In *LICS* [9], pages 541–550.
doi:10.1109/LICS.2012.64.

André Platzer.
Differential-algebraic dynamic logic for differential-algebraic programs.
J. Log. Comput., 20(1):309–352, 2010.
doi:10.1093/logcom/exn070.

André Platzer and Edmund M. Clarke.

Computing differential invariants of hybrid systems as fixedpoints.
Form. Methods Syst. Des., 35(1):98–120, 2009.
Special issue for selected papers from CAV'08.
doi:10.1007/s10703-009-0079-8.

André Platzer.
The structure of differential invariants and differential cut elimination.
Logical Methods in Computer Science, 8(4):1–38, 2012.
doi:10.2168/LMCS-8(4:16)2012.

André Platzer.
A differential operator approach to equational differential invariants.
In Lennart Beringer and Amy Felty, editors, *ITP*, volume 7406 of *LNCS*, pages 28–48. Springer, 2012.
doi:10.1007/978-3-642-32347-8_3.

André Platzer.
Logical Analysis of Hybrid Systems: Proving Theorems for Complex Dynamics.
Springer, Heidelberg, 2010.
doi:10.1007/978-3-642-14509-4.

Proceedings of the 27th Annual ACM/IEEE Symposium on Logic in Computer Science, LICS 2012, Dubrovnik, Croatia, June 25–28, 2012. IEEE, 2012.